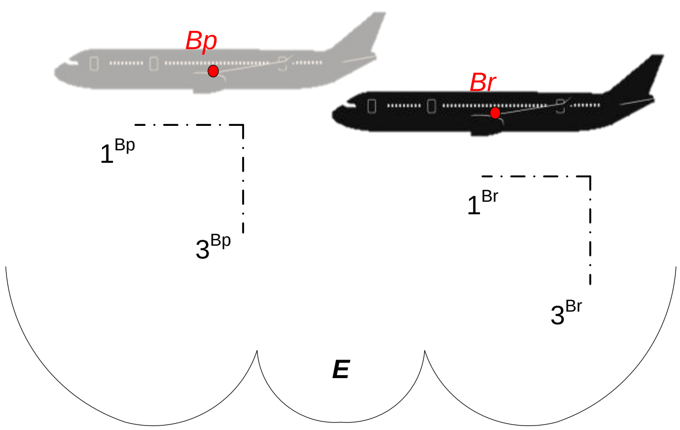
Solutions

Modeling Flight Dynamics with Tensors

Lecture 10

Problem 1 Comparison of Perturbations

While an aircraft with c.m. B_r flies in a steady horizontal flight with $[\overline{v_{Br}^E}]^{Br} = [V_r \quad 0 \quad 0]$, its velocity is perturbed by a vertical gust $[\overline{\Delta v}]^{Bp} = [0 \quad 0 \quad -\Delta V]$. Express the perturbed flight both in total and component perturbations.



Solution

Total Perturbations

Tensor relationship $\mathbf{v}_{Bp}^E = \mathbf{v}_{Br}^E + \mathbf{\Delta v}$

Matrix relationship $[v_{Bp}^E]^{Bp} = [T]^{BpBr}[v_{Br}^E]^{Br} + [\Delta v]^{Bp}$

Matrices $[v_{Bp}^E]^{Bp} = [T]^{BpBr}\begin{bmatrix} V_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\Delta V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} V_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\Delta V \end{bmatrix} = \begin{bmatrix} V_r \\ 0 \\ -\Delta V \end{bmatrix}$

Component Perturbations

Tensor relationship $\mathbf{v}_{Bp}^E = \mathbf{R}^{BpBr}\mathbf{v}_{Br}^E + \mathbf{\Delta v}$

Matrix relationship $[v_{Bp}^E]^{Bp} = [R^{BpBr}]^{Bp}[T]^{BpBr}[v_{Br}^E]^{Br} + [\Delta v]^{Bp} = [E][v_{Br}^E]^{Br} + [\Delta v]^{Bp}$

Where $[R^{BpBr}]^{Bp}[T]^{BpBr} = [E]$ (see Slide 4.6: $[R^{BA}]^{A} = [R^{BA}]^{B} = [\overline{T}]^{BA}$)

Matrices $[v_{Bp}^{E}]^{Bp} = [E]\begin{bmatrix} V_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\Delta V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} V_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\Delta V \end{bmatrix} = \begin{bmatrix} V_r \\ 0 \\ -\Delta V \end{bmatrix}$

Comment: In this simple case there is no difference between the total and component perturbation. However, in more realistic situations the transformation matrix $[T]^{BpBr}$ is not a unit matrix and is more difficult to establish.

Problem 2 Aerodynamic Perturbations

The aerodynamic forces and moments are given in body coordinates:

$[\overline{f_a}]^{B} = \begin{bmatrix} X & Y & Z \end{bmatrix}$, $[\overline{m_a}]^{B} = \begin{bmatrix} L & M & N \end{bmatrix}$. Use the functional perturbation of Slide 6 to express the force and moment perturbations in component form.

Solution

Forces

Tensors: $\varepsilon \mathbf{f}_a = \mathbf{f}_{ap} - \mathbf{R}\mathbf{f}_{ar}$

Matrices: $[\varepsilon f_a]^{Bp} = [f_{ap}]^{Bp} - \underbrace{[R^{BpBr}]^{Bp}[T]^{BpBr}}_{[E]}[f_{ar}]^{Br}$

Components: $\begin{bmatrix} \varepsilon X \\ \varepsilon Y \\ \varepsilon Z \end{bmatrix} = \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} - \begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix}$

Moments

Tensors: $\varepsilon \mathbf{m}_a = \mathbf{m}_{ap} - \mathbf{R}\mathbf{m}_{ar}$

Matrices: $[\varepsilon m_a]^{Bp} = [m_{ap}]^{Bp} - \underbrace{[R^{BpBr}]^{Bp}[T]^{BpBr}}_{[E]}[m_{ar}]^{Br}$

Components: $\begin{bmatrix} \varepsilon L \\ \varepsilon M \\ \varepsilon N \end{bmatrix} = \begin{bmatrix} L_p \\ M_p \\ N_p \end{bmatrix} - \begin{bmatrix} L_r \\ M_r \\ N_r \end{bmatrix}$

Comment: The implementation is just like Etkin's *scalar* perturbations, however here the perturbations are formulated in invariant tensor form, which enables us in the next lecture to derive the *general perturbation equations* independent of coordinate systems.

Problem 3 Neglecting Terms of Small Order

On Slide 8, before the tensor forms of the translational and attitude equations-of-motion are converted to matrices, the equations have been simplified -- not shown -- by neglecting small terms of order two $\mathcal{O}^2$. You are to find these terms and justify their being neglected.

Solution

Translational Equations $m\left(D^{Bp}\varepsilon\mathbf{v}_B^I + \underline{\mathbf{\Omega}^{BpBr}\varepsilon\mathbf{v}_B^I} + \underline{\varepsilon\mathbf{\Omega}^{BI}\mathbf{R}^{BpBr}\mathbf{v}_{Br}^I}\right) = \varepsilon\mathbf{f}_a + \varepsilon\mathbf{f}_t + \left(\mathbf{E} - \mathbf{R}^{BpBr}\right)\mathbf{f}_{gr}$

Term 2: $\underline{\mathbf{\Omega}^{BpBr}\varepsilon\mathbf{v}_B^I}$ is $\mathcal{O}^2$**,** because both $\mathbf{\Omega}^{BpBr}$ *and* $\varepsilon\mathbf{v}_B^I$ are small of $\mathcal{O}^1$.

Term 3: $\underline{\varepsilon\mathbf{\Omega}^{BI}\mathbf{R}^{BpBr}\mathbf{v}_{Br}^I}$ **,** where $\varepsilon\mathbf{\Omega}^{BI} = \mathbf{\Omega}^{BpI} - \mathbf{R}^{BpBr}\mathbf{\Omega}^{BrI}\overline{\mathbf{R}}^{BpBr}$ **,** last term is $\mathcal{O}^2$,

$$\text{therefore } \varepsilon\mathbf{\Omega}^{BI}\mathbf{R}^{BpBr}\mathbf{v}_{Br}^I = \left(\mathbf{\Omega}^{BpBr} + \underbrace{\mathbf{\Omega}^{BrI}}_{0}\right)\mathbf{R}^{BpBr}\mathbf{v}_{Br}^I = \mathbf{\Omega}^{BpBr}\mathbf{R}^{BpBr}\mathbf{v}_{Br}^I$$

Attitude Equations $\mathbf{I}_{Bp}^{Bp}D^{Bp}\boldsymbol{\omega}^{BpBr} + \mathbf{\Omega}^{BpBr}\mathbf{I}_{Bp}^{Bp}\boldsymbol{\omega}^{BpBr} = \varepsilon\mathbf{m}_a + \varepsilon\mathbf{m}_t$

Term2 $\mathbf{\Omega}^{BpBr}\mathbf{I}_{Bp}^{Bp}\boldsymbol{\omega}^{BpBr}$ is small $\mathcal{O}^2$ because $\boldsymbol{\omega}^{BpBr}$ is small of $\mathcal{O}^1$.