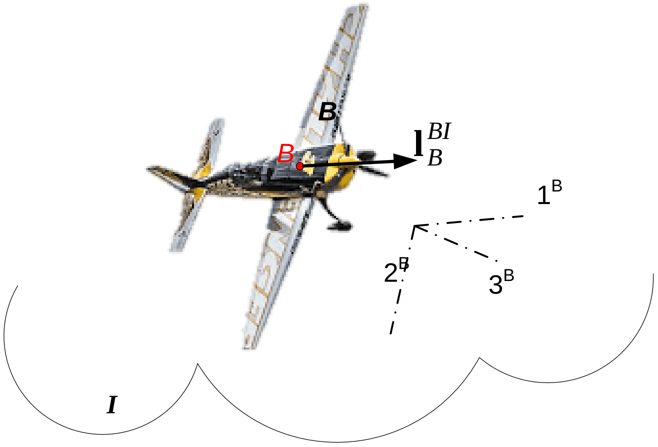
Solutions

Modeling Flight Dynamics with Tensors

Lecture 9

Problem 1 Gyroscopic Moment of Aircraft Engine

Aircraft ***B*** makes a vertical pull up at angular velocity ω with its propeller having an angular momentum l. The propeller acts like a gyro, which will give rise to a moment about the 1^3 axis. To counteract the momentum the pilot has to deflect the rudder. Which pedal will he have to press?



Solution

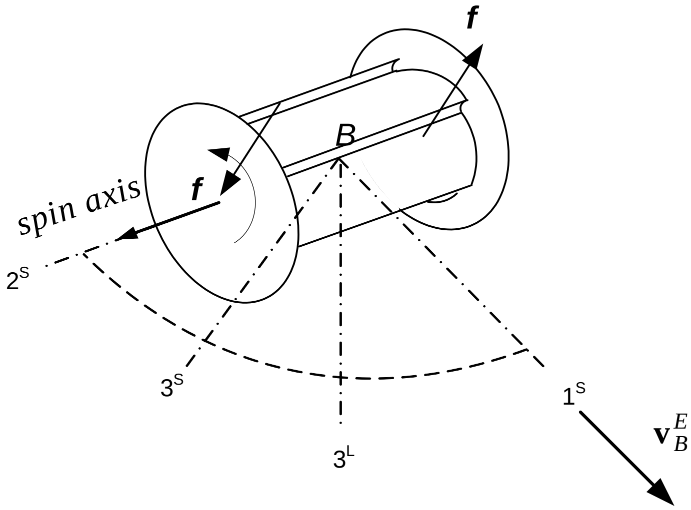
From Slide 4: $\mathbf{\Omega}^{PI}\,\mathbf{l}_B^{BI} = \mathbf{m}_B$ expressed in body coordinates $\begin{bmatrix} 0 & 0 & \omega \\ 0 & 0 & 0 \\ -\omega & 0 & 0 \end{bmatrix}\begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\omega l \end{bmatrix}$

The gyroscopic moment acts in the negative sense about the 3^B axis, so the pilot has to press the right pedal.

Problem 2 Nutation of Magnus Rotor

A Magnus Rotor is an autorotating shape, spinning about its 2^S axis, which uses the Magnus lift effect to glide to the ground with velocity $\mathbf{v}_B^E$. I investigated these Magnus rotors for my dissertation. To determine the nutational damping effect, two explosive charges were fired during descent, creating the force couple ***f***-***f***, which induced a nutation. From cinetheodolite tracking data the nutational damping was then derived.

Compute the initial value of the nutation angle θ. The numerical values are $m_B = 2$ *Nm*, $\Delta t = 0.5$ *sec*, $I = 0.0268$ *kg m*^{2}, $\omega = 1000$ *RPM*.



Solution

From Slide 5: Nutation Angle $\theta = \arctan\left(\frac{|m_B|}{|l_0|}\Delta t\right) = \arctan\left(\frac{2}{3.94}0.5\right) = 14.24\,deg$

Problem 3 Energy Theorem Derivation

Slide 7 states the *energy theorem* $\frac{dT^{BI}}{dt} = \overline{\boldsymbol{\omega}}^{BI}\mathbf{m}_B$, i.e., how the external moment changes the kinetic energy. Derive the theorem starting with the rotary part of the previous equation.

Solution

Let us assume that point *B* is fixed in the inertial frame $\boldsymbol{I}$**,** so that we can concentrate on the rotational kinetic energy. From Slide 7

$$2T^{BI} = \overline{\boldsymbol{\omega}}^{BI}\mathbf{I}_B^B\,\boldsymbol{\omega}^{BI}$$

Take the time derivative of the *scalar* T^{BI}, which is equivalent to the rotational derivative wrt any frame, and specifically the body frame. Then apply the chain rule

$$2\frac{dT^{BI}}{dt} = 2D^BT^{BI} = \overline{D^B\boldsymbol{\omega}^{BI}}\,\mathbf{I}_B^B\,\boldsymbol{\omega}^{BI} + \overline{\boldsymbol{\omega}}^{BI}D^B\left(\mathbf{I}_B^B\;\boldsymbol{\omega}^{BI}\right)$$

Recognize that the *first* term on the right equals the second term, because (1) the term is a scalar, and (2) the body ***B*** is rigid, which enables us to move $\mathbf{I}_B^B$ (symmetric tensor) under the rotational derivative

$$\overline{D^B\omega^{BI}}\,\mathbf{I}_B^B\,\boldsymbol{\omega}^{BI} = \overline{\boldsymbol{\omega}}^{BI}\bar{\mathbf{I}}_B^B\,D^B\boldsymbol{\omega}^{BI} = \overline{\boldsymbol{\omega}}^{BI}D^B\left(\mathbf{I}_B^B\;\boldsymbol{\omega}^{BI}\right)$$

Thus, introducing the angular momentum $\mathbf{l}_B^{BI} = \mathbf{I}_B^B\,\boldsymbol{\omega}^{BI}$ we get

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$$\frac{dT^{BI}}{dt} = \overline{\boldsymbol{\omega}}^{BI}D^B\left(\mathbf{I}_B^B\boldsymbol{\omega}^{BI}\right) = \overline{\boldsymbol{\omega}}^{BI}D^B\mathbf{l}_B^{BI}$$

To replace the angular momentum term by the external moment, we substitute Euler's equations, transformed to the body frame $D^B\mathbf{l}_B^{BI} + \boldsymbol{\Omega}^{BI}\mathbf{l}_B^{BI} = \mathbf{m}_B$. Since the cross product with the same vectors vanishes, the proof is completed.