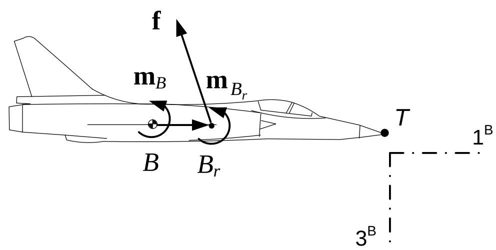
Solutions

Modeling Flight Dynamics with Tensors

Lecture 8

Problem 1 Shift of Aerodynamic Reference Point



Slide 6 provides the equation to shift the aerodynamic forces and moments from the wind tunnel reference B_r to the center of mass B : $\mathbf{m}_B = \underbrace{\mathbf{m}_{B_r}}_{free\ moment} + \underbrace{\mathbf{S}_{B_rB}\mathbf{f}}_{torque\ wrt\ B}$ The aerodynamic forces are free and can be applied directly to the c.m. However, the moments must be adjusted for the shift. The forces and moments are measured in the wind tunnel with respect to B_r and provided in body coordinates $[\overline{f}]^B = \begin{bmatrix} X & Y & Z \end{bmatrix}$, $[\overline{m}]^B = \begin{bmatrix} L & M & N \end{bmatrix}$. Provide the correction terms in body coordinates. Note that by convention the points B and B_r are measured from the tip of the aircraft T (or missile) backwards positive and for our simplified approach both points lie on the 1^B axis.

Solution

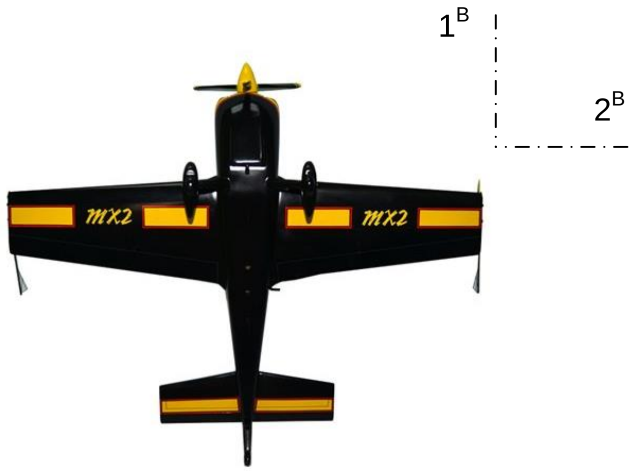
Equation from Slide 6 in body coordinates $[m_B]^B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -(s_{BrB})_1^B \\ 0 & (s_{BrB})_1^B & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

But because the points B and B_r are measured from the tip of the aircraft positive in the negative direction of the 1^B axis we must replace $(s_{BrB})_1^B = -(s_{BrB})_1^B$

to get the aerodynamic moment adjustments

$$[m_B]^B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & (s_{BBr})_1^B \\ 0 & -(s_{BBr})_1^B & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} + \begin{bmatrix} 0 \\ Z \cdot (s_{BrB})_1^B \\ -Y \cdot (s_{BrB})_1^B \end{bmatrix}$$

Problem 2 Pull-Up in Non-Inertial Frame



Slide 8 gives the correction factors if Euler's law is referred to a non-inertial reference frame. You are to determine their magnitudes for an aircraft that makes a vertical pull-up from horizontal at the constant pitch rate q = 0.5 *rad/sec* with respect to the non-inertial Earth frame E. For ease of calculation let the aircraft start at longitude l = 0 *deg*, latitude λ= 0 *deg*, altitude h = 0 *m* and neglect their changes. The moment of inertia matrix of the aircraft is

$$[I_B^B]^B = \begin{bmatrix} 6437 & 0 & 0 \\ 0 & 37836 & 0 \\ 0 & 0 & 42775 \end{bmatrix} kgm^2$$

Run for three seconds.

Solution

TENSORS

From Slide 8: $D^E\left(\mathbf{I}_B^B\boldsymbol{\omega}^{BE}\right)=\mathbf{m}_B-\underbrace{\underbrace{D^E\left(\mathbf{I}_B^B\boldsymbol{\omega}^{EI}\right)}_{rate-of-change}-\underbrace{\boldsymbol{\Omega}^{EI}\mathbf{I}_B^B\boldsymbol{\omega}^{BI}}_{precession}}_{error\ terms}$

First Correction

$$D^E\left(\mathbf{I}_B^B\boldsymbol{\omega}^{EI}\right)=D^E\left(\mathbf{I}_B^B\right)\boldsymbol{\omega}^{EI}+\mathbf{I}_B^B\underbrace{D^E\left(\boldsymbol{\omega}^{EI}\right)}_{0}=\left(\underbrace{D^B\left(\mathbf{I}_B^B\right)}_{0}+\Omega^{BE}\mathbf{I}_B^B+\mathbf{I}_B^B\overline{\Omega}^{BE}\right)\boldsymbol{\omega}^{EI}$$

$$=\left(\Omega^{BE}\mathbf{I}_B^B+\mathbf{I}_B^B\overline{\Omega}^{BE}\right)\boldsymbol{\omega}^{EI}$$

Note: Euler's law for tensors of rank two: $D^A\mathbf{X} = D^B\mathbf{X} + \mathbf{\Omega}^{BA}\mathbf{X} + \mathbf{X}\overline{\mathbf{\Omega}}^{BA}$

Ref: Zipfel, "Tensors of Rank Two in Tensor Flight Dynamics", Advances in Aerospace Science and Technology, 2018

Second Correction

$$\mathbf{\Omega}^{EI}\mathbf{I}_B^B\boldsymbol{\omega}^{BI} = \mathbf{\Omega}^{EI}\mathbf{I}_B^B\left(\boldsymbol{\omega}^{BE} + \boldsymbol{\omega}^{EI}\right)$$

MATRICES

First Correction: $[First]^B = \left[D^E\left(\mathbf{I}_B^B\boldsymbol{\omega}^{EI}\right)\right]^B = \left([\Omega^{BE}]^B[\mathrm{I}_B^B]^B + [\mathrm{I}_B^B]^B[\overline{\Omega}^{BE}]^B\right)[T]^{BE}[\omega^{EI}]^E$

Second Correction: $[Second]^B = \left[\mathbf{\Omega}^{EI}\mathbf{I}_B^B\boldsymbol{\omega}^{BI}\right]^B = [T]^{BE}[\Omega^{EI}]^E[\bar{T}]^{BE}[\mathrm{I}_B^B]^B\left([\omega^{BE}]^B + [T]^{BE}[\omega^{EI}]^E\right)$

Details:

$[\overline{\omega}^{BE}]^B = \begin{bmatrix}0 & q & 0\end{bmatrix}$, $q = 0.5\ rad/\sec$, $[\overline{\omega}^{EI}]^B = \begin{bmatrix}0 & 0 & \omega^{EI}\end{bmatrix}^E$, $\omega^{EI} = 7.292115\text{e-}5\ rad/\sec$

$$[T]^{BE} = [T]^{BG}[T]^{GE} = \begin{bmatrix} 0 & 0 & \cos(q\cdot t) \\ 0 & 1 & 0 \\ -\cos(q\cdot t) & 0 & 0 \end{bmatrix}$$

SCILAB Results

```
"First Correction - kg*m^2/sec^2"
1.1627130
0.7158489
0.0937200
0.5513548
"Second Correction - kg*m^2/sec^2"
1.2106447
0.7453591
0.0975835
0.5740838
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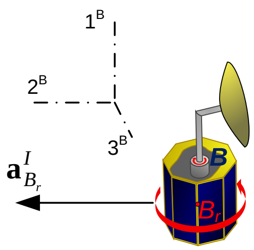
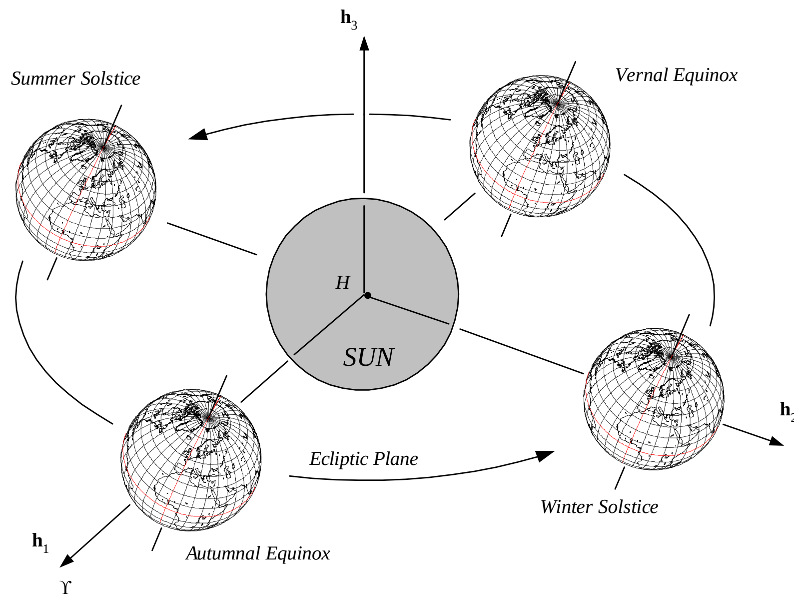
Problem 3 Flying Towards Mars





A spin-stabilized satellite with a non-spinning antenna and propulsion system is flying towards Mars. As it leaves the Earth vicinity it is spun-up and propelled towards Mars. To solve the attitude dynamics, we use Euler's law not wrt the overall center-of-mass *B*, but wrt the center of the main body B_r. Calculate the correction factor if the total satellite mass is $m^B = 1000\ kg$, the displacement of the of the reference points is in the non-spinning body coordinates $[s_{BBr}]^B = [0.5 \quad -0.2 \quad 0]$ *m*, and the acceleration due to the propulsion $[a^I_{B_r}]^B = [0 \quad 0.01 \quad 0]\ m/s^2$.

Solution

The equation of Slide 9 $D^I\left(\mathbf{I}^B_{B_r}\,\boldsymbol{\omega}^{BI}\right) = \mathbf{m}_{B_r} - m^B \mathbf{S}_{BB_r} D^I D^I \mathbf{s}_{B_r I}$ provides the correction factor

$$[correction]^B = -m^B [S_{BB_r}]^B [D^I D^I \mathbf{s}_{B_r I}]^B = -m^B [S_{BB_r}]^B [a^I_B]^B$$

$$[correction]^B = -1000 \begin{bmatrix} 0 & 0 & -0.2 \\ 0 & 0 & -0.5 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} kgm^2/s^2$$