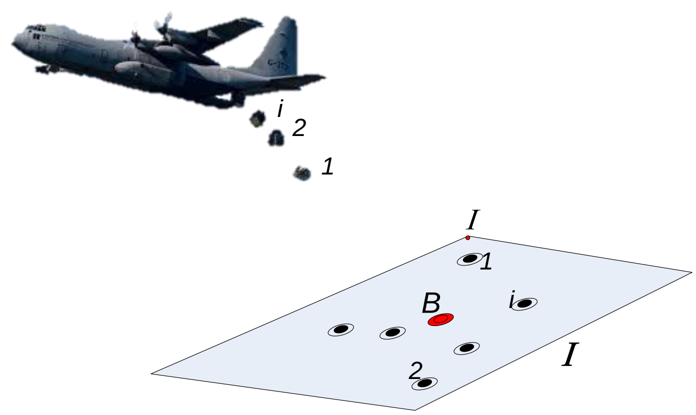
Solutions

Modeling Flight Dynamics with Tensors

Lecture 6

Problem 1 Centroid of Delivery Pattern

The Air Force conducts tests to determine the release conditions for cargo pallets. The criteria is the centroid *B* of the impact patterns with respect to a reference point I of the ground frame I. The individual pallets are equipped with accelerometers and telemetry. Use Slide 5 to give the test crew some guidance how to determine $\mathbf{s}_{BI}$. Assume that the pallets can be modeled as point-mass with center-of-mass *i*.



Solution

With the pallets modeled as point-mass, they can be treated like particles. From Slide 5 we have $\sum_i m_i D^I D^I \mathbf{s}_{iI} = \sum_i \mathbf{f}_i$ and $\sum_i m_i D^I D^I \mathbf{s}_{BI} = \sum_i \mathbf{f}_i$. Setting them equal

$$\sum_i m_i D^I D^I \mathbf{s}_{BI} = \sum_i m_i D^I D^I \mathbf{s}_{iI}$$

$$\mathbf{s}_{BI} = \frac{1}{m^{\Sigma}} \sum_i m_i \iint D^I D^I \mathbf{s}_{iI} dt dt$$

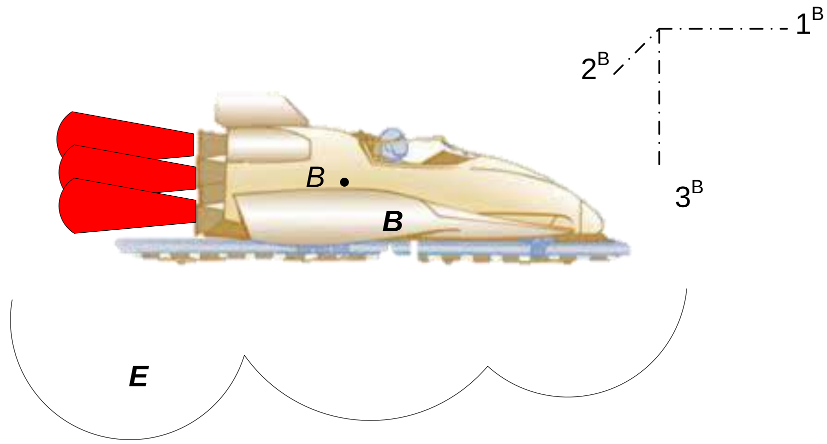
$$\mathbf{s}_{BI} = \frac{1}{m^{\Sigma}} \sum_i m_i \iint \mathbf{a}_i^I dt dt$$

$\mathbf{a}_i^I$ are the accelerometer measurements. This calculation has to be executed from launch until impact.

Comment: You will say there is a much simpler approach: Just analyze the impact points to get the centroid. True, but what if the delivery is over water?

Problem 2 Coriolis Transformation

You are racing down the Salt Flats at Bonneville Racetrack, Uta at you max speed of 600 *MPH* going North. What Coriolis and centrifugal accelerations do you experience? Can you feel them?



Solution

Slide 7 provides the Coriolis and centrifugal terms $m^B D^E \mathbf{v}_B^E = \mathbf{f} - m^B\left(2\boldsymbol{\Omega}^{EI}\mathbf{v}_B^E + \boldsymbol{\Omega}^{EI}\boldsymbol{\Omega}^{EI}\mathbf{s}_{BE}\right)$

Coriolis acceleration in body coordinates $[a_{Cor}]^B = -2[T]^{BE}[\Omega^{EI}]^E[\bar{T}]^{BE}[v_B^E]^B$

Centrifugal acceleration in body coordinates $[a_{Cent}]^B = -[T]^{BE}[\Omega^{EI}]^E[\Omega^{EI}]^E[\bar{T}]^{BE}[s_{BE}]^B$

Details: $[\overline{v_B^E}]^B = \begin{bmatrix}268.2 & 0 & 0\end{bmatrix}$ *m/s*, $[\overline{s_{BE}}]^B = \begin{bmatrix}0 & 0 & 6379554\end{bmatrix}$ *m* (Bonneville alt =1554 *m*)

$$[\Omega^{EI}]^E = \begin{bmatrix} 0 & -\omega^{EI} & 0 \\ \omega^{EI} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \omega^{EI} = 1.99\times 10^{-7} \textit{ rad/sec}, \ [T]^{BG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

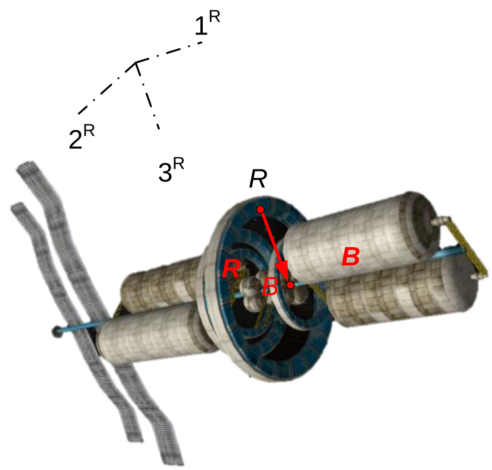
$$[T]^{BE} = [T]^{BG}[T]^{GE} = \begin{bmatrix} -\sin\lambda\cos l & -\sin\lambda\sin l & \cos\lambda \\ -\sin l & \cos l & 0 \\ -\cos\lambda\cos l & -\cos\lambda\sin l & -\sin\lambda \end{bmatrix} \quad \lambda = 0.712 \textit{ rad}, \ l = -1.986 \textit{ rad}$$

Here is my SCILAB result. The major Coriolis acceleration is to the East, but not noticeable.

```
"Coriolis Acceleration- m/s^2"
0.0000085
0.0000640
0.0000098
"Centrifugal Acceleration - m/s^2"
6.217D-08
-1.007D-23
7.203D-08
```

Problem 3 Grubin Transformation

You are journeying to Mars, cruising at constant speed. Your spaceship ***B*** has the capability to spin up its central rotational part ***R*** to provide you, located at *R*, with artificial gravity. Your distance from the c.m. of the spaceship *B* is in $]^R$ coordinates $[\overline{s_{BR}}]^R = [30 \quad 0 \quad 100]\ m$ and the spin-up rate is $\dot{\omega}^{RB} = 0.0001\ rad/s^2$. How long do you have to wait until you experience the nominal Earth gravitational acceleration?. . You may neglect the start-up inertia transient.



Solution

Slide 8: $m^B D^I \mathbf{v}_{B_r}^I = \mathbf{f} - m^B \left(\underbrace{\mathbf{\Omega}^{BI}\mathbf{\Omega}^{BI}\mathbf{s}_{BB_r}}_{Centrifugal\ accel.} + \underbrace{\left(D^I\mathbf{\Omega}^{BI}\right)\mathbf{s}_{BB_r}}_{Angular\ accel.} \right)$ Without external forces the mass terms cancel and furthermore the spaceship frame ***B*** is an inertial frame ***I*** because it cruises at constant speed. Replacing therefore frame ***I*** by ***B*** and ***B*** by ***R*** and with point B_r now *R*, we get $D^B\mathbf{v}_R^B = \mathbf{\Omega}^{RB}\mathbf{\Omega}^{RB}\mathbf{s}_{BR} + \left(D^B\mathbf{\Omega}^{RB}\right)\mathbf{s}_{BR}$. The left side is the acceleration you experience. Furthermore, we can shift the rotational derivative to frame ***R*** $D^B\mathbf{\Omega}^{RB} = D^R\mathbf{\Omega}^{RB}$ (see Slide 5.12 Property 3, which also holds for the skew-symmetric part of a vector). Then we have the result in tensor form

$$\mathbf{a}_R^B = \mathbf{\Omega}^{RB}\mathbf{\Omega}^{RB}\mathbf{s}_{BR} + \left(D^R\mathbf{\Omega}^{RB}\right)\mathbf{s}_{BR}$$

Introducing the $]^R$ coordinate system

$$[a_R^B]^R = \left([\Omega^{RB}]^R \Omega^{RB}]^R + \left[\frac{d\,\Omega^{RB}}{dt}\right]^R \right)[s_{BR}]^R$$

With $[\Omega^{RB}]^R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\omega \\ 0 & \omega & 0 \end{bmatrix}$, $[\dot{\Omega}^{RB}]^R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\dot{\omega} \\ 0 & \dot{\omega} & 0 \end{bmatrix}$, $[s_{BR}]^R = \begin{bmatrix} 30 \\ 0 \\ 100 \end{bmatrix}$

Now it's time again to go to SCILAB. Here is the result:

```
"Time="

 52.216667

"min"

"Acceleration="

 9.8156941

"m/s^2"
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