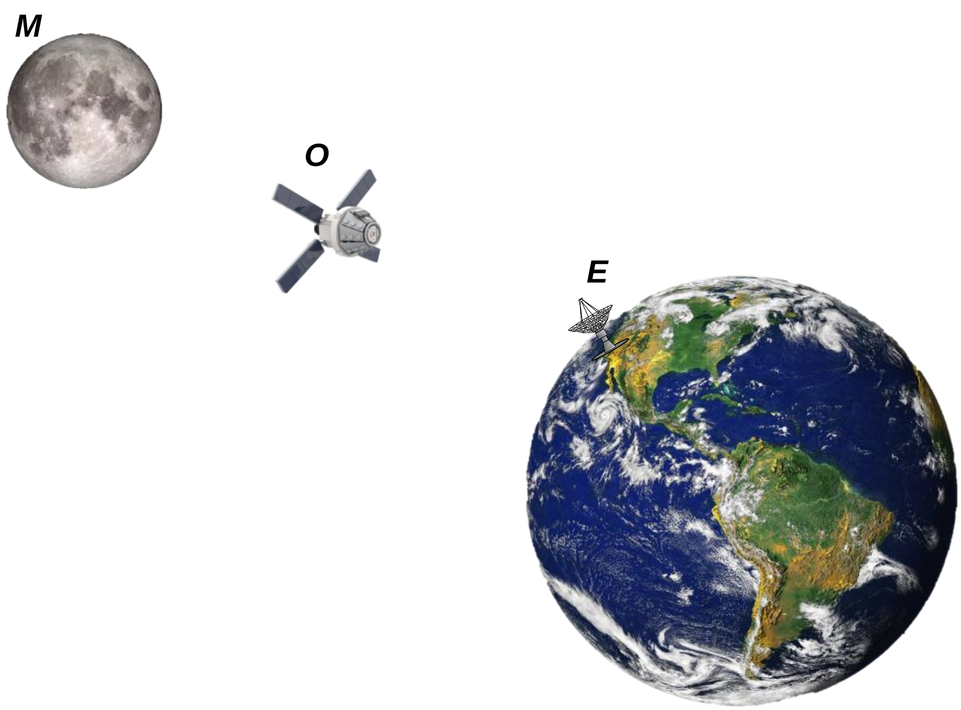
Solutions

Modeling Flight Dynamics with Tensors

Lecture 4

Problem 1 Rotation Tensors

To land the Orion on the Moon it will be oriented by the radar signals from Vandenberg. Known are the relative orientations given by rotation tensors $\boldsymbol{R}^{ME}$ and $\boldsymbol{R}^{OE}$. Express the orientation of the Orion wrt the moon in terms of the rotation tensor $\boldsymbol{R}^{OM}$.



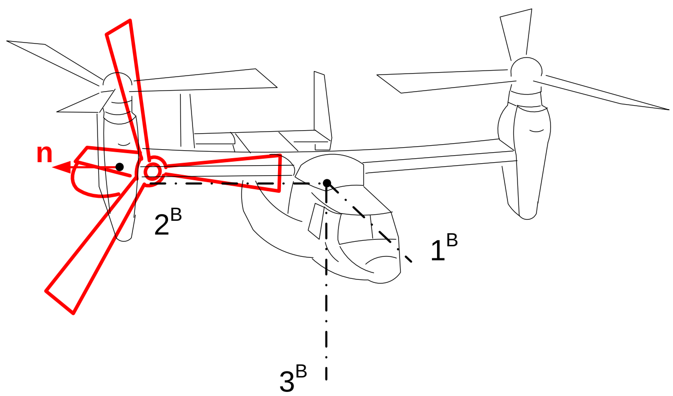
Solution

I follow the instructions given in Slide 5: $\mathbf{R}^{OM} = \mathbf{R}^{OE}\overline{\mathbf{R}^{ME}}$

Comment: The situation is modeled solely by second order tensors. Only for actual computations do we have to invoke coordinate systems.

Problem 2 Rotation Vector

The Osprey rotates its propellers from vertical take-off to horizontal flight through 90 *deg*. The rotation tensor of the right propeller is in body coordinates $[R_{90}]^B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. Determine the unit rotation vector $[n]^B$ using the equations of Slide 9.



Solution

From Slide 9:

$$\psi = \arccos\left(\frac{1}{2}\sum_{i=1}^{3} r_{ii} - \frac{1}{2}\right) = \arccos\left(\frac{1}{2}\times 1 - \frac{1}{2}\right) = \arccos 0 = 90^o$$

$$[n]^B = \frac{1}{2\sin\psi}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix} = \frac{1}{2\sin 90^o}\begin{bmatrix} 0-0 \\ 1+1 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/\sin 90^o \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Problem 3 180^0 Rotation Tensor

Use the general rotation tensor equation of Slide 7 and derive the tensor $\boldsymbol{R}_{180}$ for a 180° rotation. You also can get it by multiplying $\boldsymbol{R}_{90}$ with itself $\mathbf{R}_{180} = \mathbf{R}_{90}\mathbf{R}_{90}$. To demonstrate that this is true use a general unit rotation vector $\boldsymbol{n}$ for your $\boldsymbol{R}_{180}$ relationship and the $\boldsymbol{R}_{90}$, and show that they both lead to the same numerical result.

Solution

From Slide 7: $\mathbf{R}_{180} = \cos 180^o \mathbf{E} + (1 - \cos 180^o)\mathbf{n}\,\overline{\mathbf{n}} + \sin 180^o \mathbf{N} = -\mathbf{E} + 2\mathbf{n}\overline{\mathbf{n}}$, compared to $\mathbf{R}_{90} = \mathbf{n}\,\overline{\mathbf{n}} + \mathbf{N}$. I use the general unit rotation vector $\overline{[n]} = [1 \quad 2 \quad 3]/|n|$. Now it's time to go to SCILAB for the number crunching (of course you can also do this with MATLAB). The result looks like this

```
"n="
0.2672612   0.5345225   0.8017837
"N="
0.        -0.8017837   0.5345225
0.8017837   0.        -0.2672612
-0.5345225   0.2672612   0.
"R180_1=R90*R90"
-0.8571429   0.2857143   0.4285714
0.2857143  -0.4285714   0.8571429
0.4285714   0.8571429   0.2857143
"R180_2=-E+n*tans(n)"
-0.8571429   0.2857143   0.4285714
0.2857143  -0.4285714   0.8571429
0.4285714   0.8571429   0.2857143
"Norm="
1.
```

Indeed, the two computations of $\boldsymbol{R}_{180}$ for a general unit rotation vector $\boldsymbol{n}$ lead to the same numerical result.

Comment: I also printed out the Norm of the rotation matrix, which is +1 just as it is for the coordinate transformation matrix, see Slide 6.