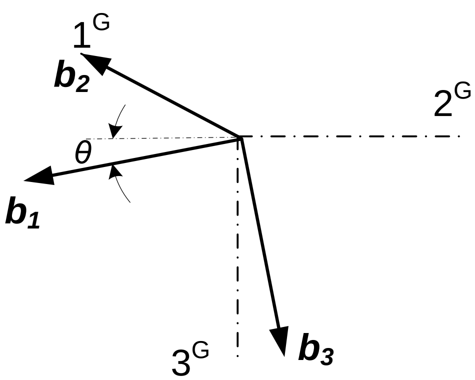
Solutions

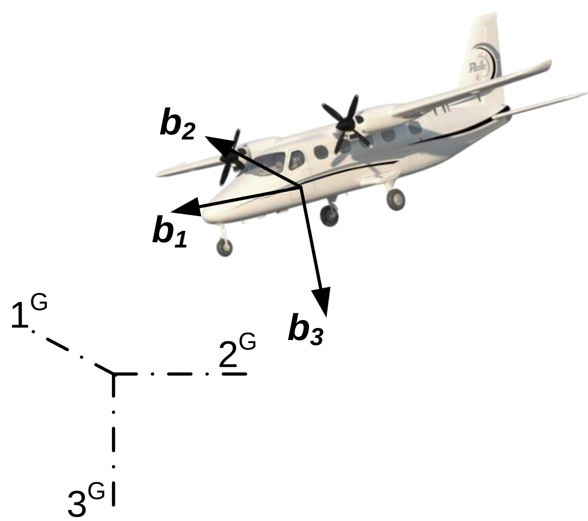
Modeling Flight Dynamics with Tensors

Lecture 3

Problem 1 Transformation Matrix

Let's use the example of Lecture 2 again of the aircraft approaching the runway. Now you are to determine the transformation matrix between the aircraft's body coordinates wrt the geographic coordinates $[T]^{BG}$ using the relationship of Slide 9 of the transformation matrix consisting of base row vectors. The aircraft's base vector $\boldsymbol{b_1}$ is tilted down by θ = 10 *deg* wrt to the horizon. (The aircraft's body axes $]^B$ are the preferred coordinate axes, overlaying its base triad $\boldsymbol{b_1}$, $\boldsymbol{b_2}$, $\boldsymbol{b_3}$.)





Solution

From Slide 9 $[T]^{BG} = \begin{bmatrix} [\overline{b_1}]^G \\ [\overline{b_2}]^G \\ [\overline{b_3}]^G \end{bmatrix}$. By inspection: $\overline{[b_1]}^G = [0 \quad -\cos\theta \quad \sin\theta]$, $\overline{[b_2]}^G = [1 \quad 0 \quad 0]$,

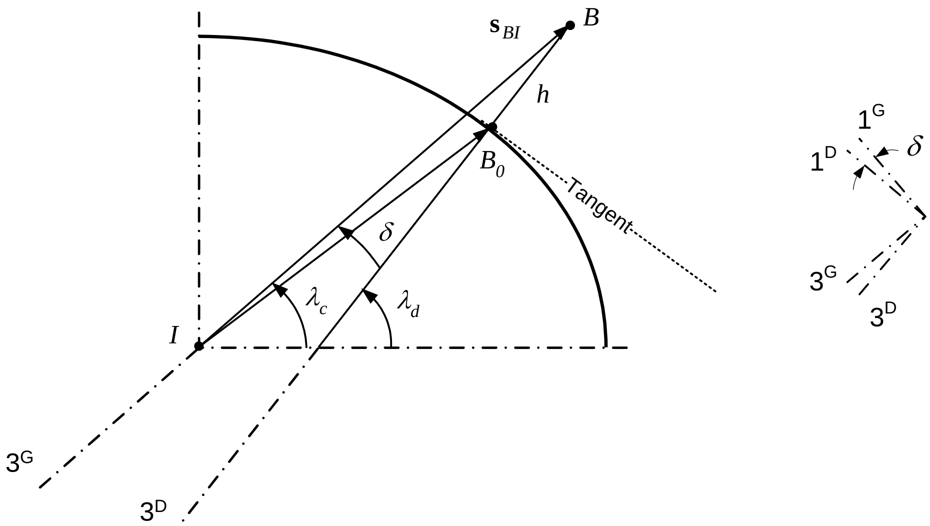
$\overline{[b_3]}^G = [0 \quad \sin\theta \quad \cos\theta]$

Therefore, the transformation matrix is $[T]^{BG} = \begin{bmatrix} [\overline{b_1}]^G \\ [\overline{b_2}]^G \\ [\overline{b_3}]^G \end{bmatrix} = \begin{bmatrix} 0 & -\cos\theta & \sin\theta \\ 1 & 0 & 0 \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$.

Problem 2 Geodetic Coordinates

With Slide 13 I introduced you to the *geocentric* coordinate system $]^{G}$, which is used when the Earth is modeled as a sphere. In this case the 3^{G} axis goes through the center of the earth. However, a more accurate Earth model is a spheroid. Here the vertical projection of point ***B*** onto the Earth's surface does not project through the center of the Earth. This axis is labeled 3^{D} with *D* standing for geo**d**etic and the *geocentric* latitude λ_c is renamed *geodetic* latitude λ_d.

You are to derive the transformation matrix $[T]^{DG}$ with the deflection angle $\delta = \lambda_d - \lambda_g$ using the *pattern-scheme of transformation matrices* explained in the lecture.



Solution

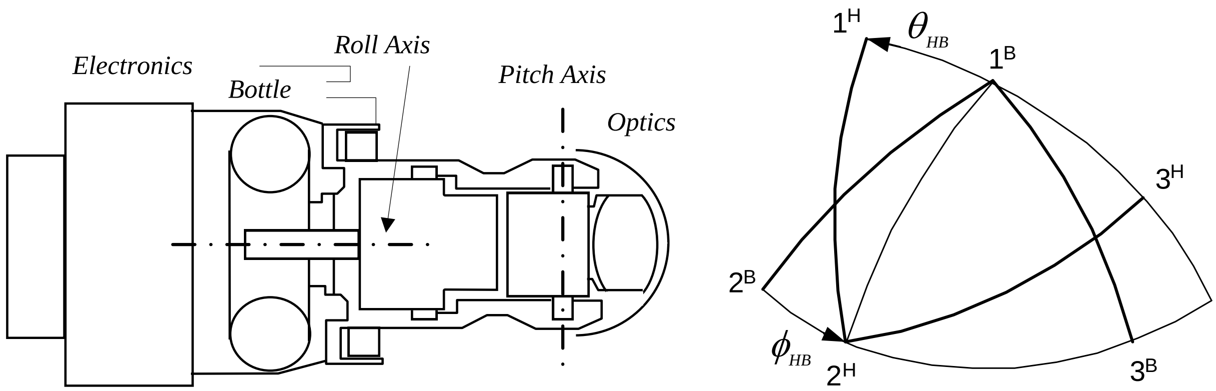
The transformation is about the 2-axis. Then, according to our pattern-scheme, of transformation matrices, the 1 is at location 2,2 and its row and column are filled in with *0*s; furthermore, the diagonal contains the cos δ and the off-diagonal elements the sin δ. The negative sign of the sin δ is below the row that contains the 1.

$$[T]^{DG} = \begin{bmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{bmatrix}$$

Comment: Notice that the longitude is not effected by the change in Earth model.

Problem 3 IR Seeker Gimbal Transformation

A typical IR sensor has two gimbal axes: *roll* and *pitch*. The roll axis allows the optics to be turned relative to the missile body, followed by the pitch axis. The outer roll gimbal angle is ϕ_{HB} and the inner pitch gimbal angle θ_{HB} . Determine the transformation matrix $[T]^{HB}$ based on the orange peel diagram on the right side.



Solution

Using your skills acquired from Lecture 3 you should get the following result:

$$[T]^{HB} = \begin{bmatrix} \cos\theta_{HB} & \sin\theta_{HB}\sin\phi_{HB} & -\sin\theta_{HB}\cos\phi_{HB} \\ 0 & \cos\phi_{HB} & \sin\phi_{HB} \\ \sin\theta_{HB} & -\cos\theta_{HB}\sin\phi_{HB} & \cos\theta_{HB}\cos\phi_{HB} \end{bmatrix}$$