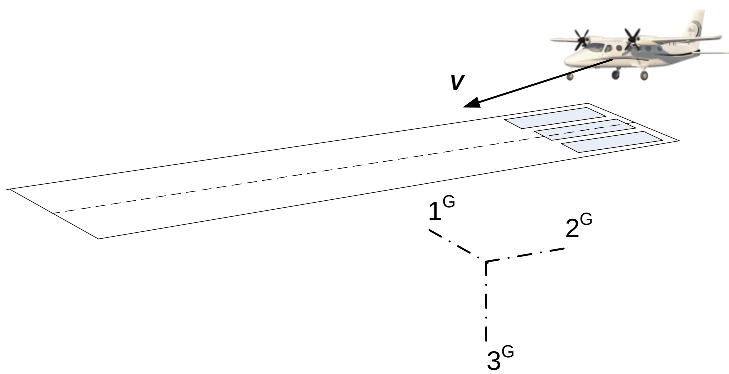
Solutions

Modeling Flight Dynamics with Tensors

Lecture 2

Problem 1 Coordinate Transformation

An airplane approaches Runway ***27*** with the speed V = 80 m/s at a glide path angle of Υ = -20 deg. What is its ground speed $[v]^G$ in the geographic coordinates of North 1^G, East 2^G, Down 3^G?



Solution

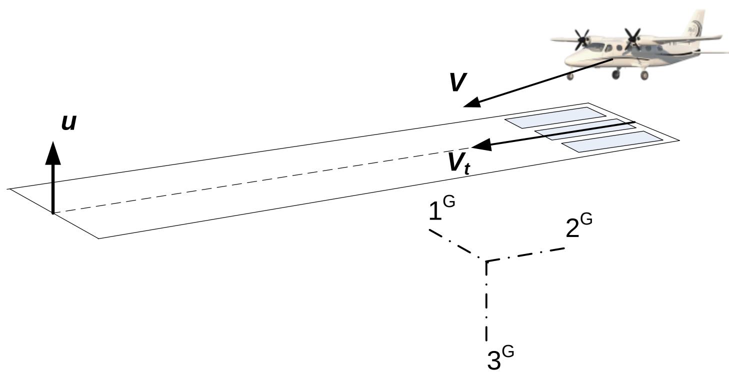
By inspection, the velocity in geographic coordinates is $\overline{[v]}^G = V[0 \quad -\cos\gamma \quad \sin\gamma]$:

$\overline{[v]}^G = 80\times[0 \quad -\cos 20 \quad \sin 20] = [0 \quad -75.2 \quad 27.4]$ m/s.

Comment: This is a simple problem. Later on for more complex transformations we will also define a velocity coordinate system $]^V$ and a transformation matrix $[T]^{GV}$; and then solve for $[v]^G = [T]^{GV}[v]^V$.

Problem 2 Plane Projection Tensor N

Use the projection tensor **N** of Slide 11 to model the projection $\boldsymbol{v_t}$ of the velocity vector $\boldsymbol{v}$ onto the landing strip, whose orientation is given by the unit vector $\boldsymbol{u}$. Then convert the tensor relationship into a matrix relationship by introducing the $]^G$ coordinate system and crunch the numbers to obtain $[v_t]^G$.



Solution

From Slide 11 $\mathbf{s} = \mathbf{N}\mathbf{t}$, where $\mathbf{N} = (\mathbf{E} - \mathbf{u}\bar{\mathbf{u}})$. In our nomenclature $\mathbf{v}_t = \mathbf{N}\mathbf{v} = (\mathbf{E} - \mathbf{u}\bar{\mathbf{u}})\mathbf{v}$. This is a tensor relationship. Now let's crunch some numbers by introducing the $]^G$ coordinate system

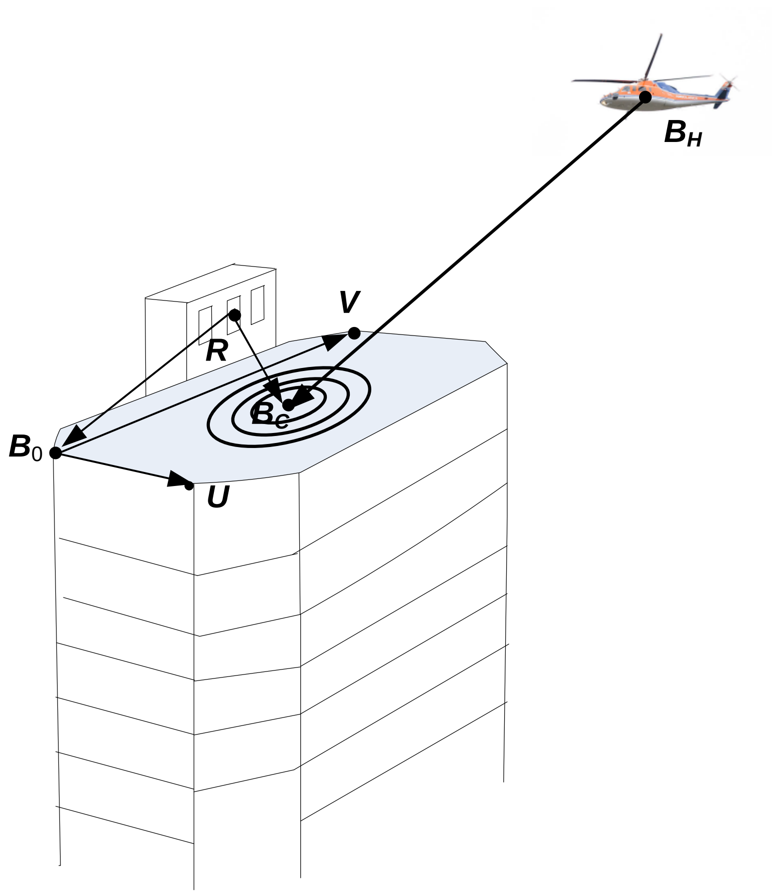
$$[v_t]^G = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \end{bmatrix} \right) \begin{bmatrix} 0 \\ -75.2 \\ 27.4 \end{bmatrix}$$

$$[v_t]^G = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} 0 \\ -75.2 \\ 27.4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -75.2 \\ 27.4 \end{bmatrix} = \begin{bmatrix} 0 \\ -75.2 \\ 0 \end{bmatrix}$$

Comment: Of course, we could have given that result just by inspecting the solution of Problem 1. But I wanted you to get comfortable with the planar projection tensor so you can apply it to more complex situations.

Problem 3 Flat Plane

On top of a tall business high-rise is centered a helipad. A helicopter ***B****H* approaches for a landing and needs to establish a glide-slope vector $\mathbf{s}_{B_C B_H}$. To determine ***B****C*, use the flat plane equation of Slide 13. You can assume that the locations of the points on the periphery of the building ***B****0*, ***U***, ***V*** are known, as well as the traffic controller's point ***R***.



Solution

From Slide 13: $\mathbf{s}_{BR} = u\mathbf{s}_{UB_0} + v\mathbf{s}_{VB_0} + \mathbf{s}_{B_0R}$. Since the Helipad is centered, $\mathbf{s}_{B_CR} = 0.5\mathbf{s}_{UB_0} + 0.5\mathbf{s}_{VB_0} + \mathbf{s}_{B_0R}$ and the glide slope vector is $\mathbf{s}_{B_CB_H} = \mathbf{s}_{B_CR} + \mathbf{s}_{RB_H}$.

Comment: The scenario is modeled entirely by tensors.