1. Question — On average the SP500 approximately yields 9% each year (since 1950) paired with a standard deviation of roughly 15%. In 2021, its yield amounted to 26.89%. What is the z-score for this value?

Answer :

$$z = \frac{x - \mu}{\sigma}$$
$$\leftrightarrow \frac{26.89 - 9}{15} = 1.193$$

2. Question — Which of the following scenarios are likely to be distributed normally? Which of them are skewed?

1. Annual stock market returns over 200 years
2. Household income of all US households
3. IQ of all humans
4. Size of 5 students
5. Size of 25000 students

Answer :

1. Normal distributed around the mean of 9%
2. Skewed normal distribution (to the right) due to very wealthy individuals
3. Normal distributed around the mean IQ of 100-105.
4. Not normal distribute, sample size too small
5. Large sample size. Bi-Normal distributed (one for women and one for man)

3. Question — Below you can find the number of defectively manufactured products of a product within the span of one month, as well as the corresponding total number of manufactured products.

Month	Errors	Total
Jan	38,123	1,000,000
Feb	35,546	900,000
Mar	41,593	1,000,000
Apr	18,555	500,000
May	39,346	1,000,000
Jun	31,008	750,000
Jul	37,993	1,000,000
Aug	44,663	1,500,000
Sep	41,221	1,200,000
Oct	37,063	850,000
Nov	42,010	1,000,000
Dec	21,925	500,000

Table 1: Erroneous and total manufactured products over one year

In a month when 1,000,000 products are manufactured, what is the probability that more than 45,000 will be defective?

Answer :

1. Compute average error rate: $\frac{1}{12}\sum_{i=1}^{12}\frac{\text{error}_i}{\text{total}_i} = 0.039$.
2. Compute std of error rate (corrected version): $\sqrt{\frac{1}{11}\sum_{i=1}^{12}(\text{error}_i - \overline{\text{error}})^2} = 0.004$.
3. Compute z-score: $\frac{x-\mu}{\sigma} = \frac{0.045-0.039}{0.004} = 1.5$
4. Look up 1.5 in z-table: $P(X \leq 0.045) = 0.9332$
5. $P(X \geq 0.045) = 1 - P(X \leq 0.045) = 1 - 0.9332 = 0.0668$

   The probability that more than 45,000 products are defective given a total of 1,000,000 is 6.68%