1. Question — Given 100 coin tosses. 40 show head and 60 tail. Can you state that P(head) = 0.4?

Answer : No, as this is the experimental probability with a small sample size.

2. Question — How often would you need to throw the coin to be certain that P(head)=0.4? How often if you just want to be "very sure"? What is the underlying rule?

Answer : Infinitely.
Maybe 10,000 times. Law of Large Numbers

3. Question — You have $10,000 to invest in three companies.
Company 1 yields 99% with a chance of 10%.
Company 2 yields 10% 99% of the time.
Company 3 yields 1000% 1% of the time.
a) What is the probability that all companies make profit?
b) What is the expected return of company 1 when investing $10,000?
c) What is the probability that no company makes profit?
d) What is the probability that only company 1 makes profit?

Answer :

a) $P(1) * P(2) * P(3) = 0.1 * 0.99 * 0.01$

b) $10,000 * P(1) * \text{yield} = 10,000 * 0.1 * 0.99 = 990$

c) $1 - P(1) * P(2) * P(3) = 1 - 0.1 * 0.99 * 0.01$

d) $P(1) * (1 - P(2)) * (1 - P(3)) = 0.1 * (1 - 0.99) * (1 - 0.01)$

4. Question — A new virus is spreading! Already 10,000 people are infected (out of 335,000,000). Luckily, a new rapid test is available which has a true positive and true negative rate of 99%. Unfortunately your test is positive now! What is the probability that you are infected?

Answer :

$$P(\text{Infected} \mid \text{positive test}) = \frac{P(\text{positive test}|\text{Infected}) * P(\text{Infected})}{P(\text{positive test})}$$

$$\leftrightarrow P(\text{Infected} \mid \text{positive test}) = \frac{0.99 * \frac{10{,}000}{335{,}000{,}000}}{P(\text{positive test})}$$

$$\leftrightarrow P(\text{Infected} \mid \text{positive test}) = \frac{0.99 * 0.00002985074}{P(\text{positive test})}$$

$$\leftrightarrow P(\text{Infected} \mid \text{positive test}) = \frac{0.00002955223}{P(\text{positive test} \mid \text{Infected}) * P(\text{Infected}) + P(\text{positive test} \mid \text{Not Infected}) * P(\text{Not Infected})}$$

$$\leftrightarrow P(\text{Infected} \mid \text{positive test}) = \frac{0.00002955223}{0.99 * 0.00002985074 + 0.01 * 0.99997014925}$$

$$\leftrightarrow P(\text{Infected} \mid \text{positive test}) = 0.00294660308 \rightarrow 0.29\%$$