Electrodynamics

SECTION

Electrodynamics in matter

LECTURE

Polarization

BÖRGE GÖBEL

Polarization

$$\rho = \rho_0 + \rho_p$$

"external" charges

Polarization charges
(due to $\boldsymbol{E}$ and $\boldsymbol{B}$ acting on matter)

$$\int \rho_p \, \mathrm{d}V = 0$$

Continuity equation

$$\dot{\rho}_p = -\nabla \cdot \boldsymbol{j}_p$$

$$\dot{\rho}_p = -\nabla \cdot \dot{\boldsymbol{P}}$$

$$\boxed{\rho_p = -\nabla \cdot \boldsymbol{P}}$$

$$J = \dot{p}$$

$$\boldsymbol{j}_P = \dot{\boldsymbol{P}}$$

$$\boxed{\boldsymbol{P} = \frac{\partial \boldsymbol{p}}{\partial V}}$$

Electric dipole density

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Magnetization

Magnetization

$$\boldsymbol{j} = \boldsymbol{j}_0 + \boldsymbol{j}_p + \boldsymbol{j}_m$$

"external" currents

$$\boldsymbol{j}_0 = \rho_0 \boldsymbol{v}$$

Polarization currents

$$\boldsymbol{j}_P = \dot{\boldsymbol{P}}$$

Circular currents
causing magnetic moments ***m***
Not related to charge separation $\dot{\rho}_m = 0$

Continuity equation $\dot{\rho} + \nabla \cdot \boldsymbol{j} = 0$

$$\dot{\rho}_0 + \nabla \cdot \boldsymbol{j}_0 + \dot{\rho}_p + \nabla \cdot \boldsymbol{j}_p + \nabla \cdot \boldsymbol{j}_m = 0$$

$$\dot{\rho}_0 + \nabla \cdot \boldsymbol{j}_0 = 0 \qquad \dot{\rho}_p + \nabla \cdot \boldsymbol{j}_p = 0$$

$$\nabla \cdot \boldsymbol{j}_m = 0$$

$$\boldsymbol{j}_m = \nabla \times \boldsymbol{M}$$

$$\boldsymbol{M} = \frac{\partial \boldsymbol{m}}{\partial V}$$

Magnetic dipole density

Polarization & Magnetization

$$\rho = \rho_0 + \rho_p$$

"external" charges

Polarization charges
(due to ***E*** and ***B*** acting on matter)

$$\rho_p = -\nabla \cdot \boldsymbol{P}$$

$$\boldsymbol{P} = \frac{\partial \boldsymbol{p}}{\partial V}$$

Electric dipole density

$$\boldsymbol{j} = \boldsymbol{j}_0 + \boldsymbol{j}_p + \boldsymbol{j}_m$$

"external" currents

Circular currents causing
magnetic moments ***m***

Polarization currents

$$\boldsymbol{j}_m = \nabla \times \boldsymbol{M}$$

$$\boldsymbol{M} = \frac{\partial \boldsymbol{m}}{\partial V}$$

Magnetic dipole density

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Maxwell's equations in matter

Polarization & Magnetization

$$\rho = \rho_0 + \rho_p$$

"external" charges

Polarization charges
(due to ***E*** and ***B*** acting on matter)

$$\rho_p = -\nabla \cdot \boldsymbol{P}$$

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Electric dipole density

$$\boldsymbol{j} = \boldsymbol{j}_0 + \boldsymbol{j}_p + \boldsymbol{j}_m$$

"external" currents

Circular currents causing
magnetic moments ***m***

Polarization currents

$$\boldsymbol{j}_m = \nabla \times \boldsymbol{M}$$

$$\boldsymbol{M} = \frac{\partial \boldsymbol{m}}{\partial V}$$

Magnetic dipole density

Maxwell‘s equations

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0\epsilon_0\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0\boldsymbol{j}$$

- Charges are sources of the electric field
- The magnetic field as no sources (no monopoles)
- Time-dependent magnetic fields generate electric fields
- Time-dependent electric fields and currents generate magnetic fields

Vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

Vacuum permittivity $\epsilon_0 = 8.854... \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$

Maxwell's equations

Maxwell

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

$$\begin{aligned} \rho &= \rho_0 + \rho_p \\ &= \rho_0 - \nabla \cdot \boldsymbol{P} \end{aligned}$$

$$\begin{aligned} \boldsymbol{j} &= \boldsymbol{j}_0 + \boldsymbol{j}_p + \nabla \times \boldsymbol{M} \\ &= \boldsymbol{j}_0 + \dot{\boldsymbol{P}} + \nabla \times \boldsymbol{M} \end{aligned}$$

$$\epsilon_0 \nabla \cdot \boldsymbol{E} = \rho_0 - \nabla \cdot \boldsymbol{P}$$

$$\frac{1}{\mu_0} \nabla \times \boldsymbol{B} = \boldsymbol{j}_0 + \dot{\boldsymbol{P}} + \nabla \times \boldsymbol{M} + \epsilon_0 \dot{\boldsymbol{E}}$$

$$\rho_0 = \nabla \cdot (\epsilon_0 \boldsymbol{E} + \boldsymbol{P})$$

$$\nabla \times \left(\frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M} \right) = \boldsymbol{j}_0 + \frac{\partial}{\partial t} (\epsilon_0 \boldsymbol{E} + \boldsymbol{P})$$

$\nabla \cdot \boldsymbol{D} = \rho_0$ with $\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$

Displacement field

$\nabla \times \boldsymbol{H} = \dot{\boldsymbol{D}} + \boldsymbol{j}_0$ with $\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M}$

magnetizing field

Maxwell's equations

vacuum

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

matter

$$\nabla \cdot \boldsymbol{D} = \rho_0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{j}_0$$

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Vacuum permittivity $\epsilon_0 = 8.854... \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$

Displacement field (with polarization) $\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$

Magnetizing field (with magnetization) $\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}$

Maxwell's equations

vacuum

$$\epsilon_0 \oint_{\partial V} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \int_V \rho \, \mathrm{d}V$$

$$\oint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\oint_{\partial A} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\int_A \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}$$

$$\frac{1}{\mu_0} \oint_{\partial A} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \int_A \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{S} + \epsilon_0 \int_A \frac{\partial \boldsymbol{E}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}$$

matter

$$\oint_{\partial V} \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = \int_V \rho_0 \, \mathrm{d}V$$

$$\oint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\oint_{\partial A} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\int_A \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}$$

$$\oint_{\partial A} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_A \boldsymbol{j}_0 \cdot \mathrm{d}\boldsymbol{S} + \int_A \frac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}$$

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