Electrodynamics

SECTION

Magnetostatics

LECTURE

Maxwell's equations

Maxwell‘s equations

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

- Charges are sources of the electric field
- The magnetic field as no sources (no monopoles)
- Time-dependent magnetic fields generate electric fields
- Time-dependent electric fields and currents generate magnetic fields

Vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7} \, \frac{\mathrm{N}}{\mathrm{A}^2}$

Vacuum permittivity $\epsilon_0 = 8.854... \cdot 10^{-12} \, \frac{\mathrm{As}}{\mathrm{Vm}}$

Magnetostatics

Differential formulation

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \cancel{\mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}} + \mu_0 \boldsymbol{j}$$

Integral formulation

$$\oint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\frac{1}{\mu_0} \oint_{\partial A} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \int_A \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{S} + \cancel{\epsilon_0 \int_A \frac{\partial \boldsymbol{E}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}}$$

Magnetostatics

Differential formulation

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \cancel{\frac{\partial \boldsymbol{E}}{\partial t}} + \mu_0 \boldsymbol{j}$$

Integral formulation

$$\oint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\frac{1}{\mu_0} \oint_{\partial A} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \int_A \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{S} + \epsilon_0 \cancel{\int_A \frac{\partial \boldsymbol{E}}{\partial t}} \cdot \mathrm{d}\boldsymbol{S}$$

Comparison electrostatics

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0} \rho$$

$$\nabla \times \boldsymbol{E} = -\cancel{\frac{\partial \boldsymbol{B}}{\partial t}}$$

$$\epsilon_0 \oint_{\partial V} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \int_V \rho \, \mathrm{d}V$$

$$\oint_{\partial A} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\int_A \cancel{\frac{\partial \boldsymbol{B}}{\partial t}} \cdot \mathrm{d}\boldsymbol{S}$$

Electrodynamics

SECTION

Magnetostatics

LECTURE

Vector potential

Vector potential

Recap: electrostatic potential in electrostatics

- Maxwell $\nabla \times \boldsymbol{E}(\boldsymbol{r}) = 0$
- We can introduce the electrostatic potential $\boldsymbol{E}(\boldsymbol{r}) = -\nabla \varphi(\boldsymbol{r})$

...Electrostatic potential can be shifted by any constant

Now: Magnetostatics

- Maxwell $\nabla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0$
- We can introduce the vector potential $\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$

...Vector potential can be shifted by any gradient of a function

$$\mathbf{A}' = \mathbf{A} + \nabla \chi$$

$$\boldsymbol{B}' = \nabla \times \boldsymbol{A}' = \nabla \times \boldsymbol{A} + \nabla \times \nabla \chi = \boldsymbol{B} + 0$$

Vector potential

- We can introduce the vector potential $\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$

- Maxwell $\nabla \times \boldsymbol{B}(\boldsymbol{r}) = \mu_0 \boldsymbol{j}(\boldsymbol{r})$

$$\nabla \times (\nabla \times \boldsymbol{A}(\boldsymbol{r})) = \mu_0 \boldsymbol{j}(\boldsymbol{r})$$

...Vector potential can be shifted by any gradient of a function

$$\mathbf{A}' = \mathbf{A} + \nabla \chi$$

- Use identity $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \Delta \boldsymbol{A}$

  and choose a gauge $\nabla \chi$ so that $\nabla \cdot \boldsymbol{A}(\boldsymbol{r}) = 0$

$$\boxed{\Delta \boldsymbol{A}(\boldsymbol{r}) = -\mu_0 \boldsymbol{j}(\boldsymbol{r})}$$

similar to Poisson equation

$$\boxed{\Delta \varphi(\boldsymbol{r}) = -\frac{1}{\epsilon_0} \rho(\boldsymbol{r})}$$

Electrodynamics

SECTION

Magnetostatics

LECTURE

Biot-Savart law

Biot-Savart law

$$\boxed{\Delta \boldsymbol{A}(\boldsymbol{r}) = -\mu_0 \boldsymbol{j}(\boldsymbol{r})}$$

Solve for all three components

similar to Poisson equation

$$\Delta \varphi(\boldsymbol{r}) = -\frac{1}{\epsilon_0}\rho(\boldsymbol{r})$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'$$

$$\boxed{\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \boldsymbol{j}(\boldsymbol{r}') \times \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}V'}$$

Biot-Savart

$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(\boldsymbol{r}')\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}V'$$

$$\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$$

$$\boldsymbol{E}(\boldsymbol{r}) = -\nabla \varphi(\boldsymbol{r})$$

Electrodynamics

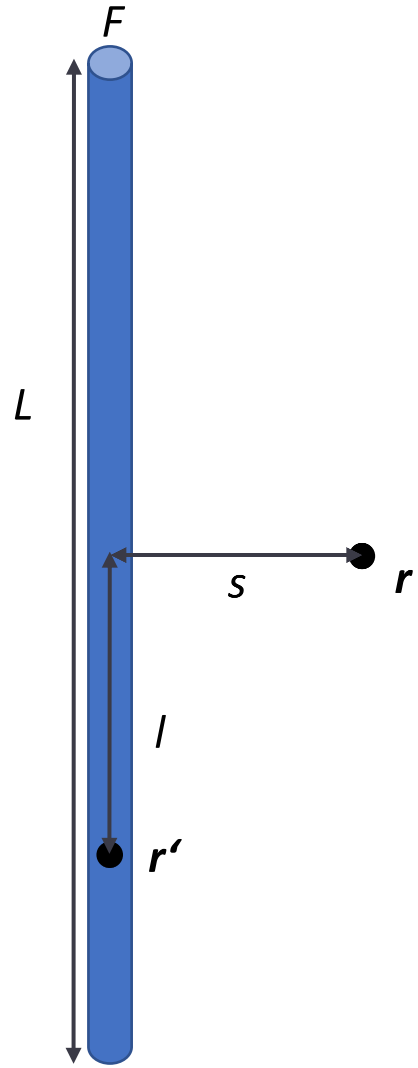
SECTION

Magnetostatics

LECTURE

Current in a wire

Charged wire



$$\boxed{\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'}$$

"Infinitely" thin wire

$|\boldsymbol{r}-\boldsymbol{r}'| \gg \sqrt{F}$

$$\int\limits_L\int\limits_F \frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}S'\mathrm{d}z' = \int\limits_L \frac{I}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}' = I\int\limits_L \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}'$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}I\int\limits_L \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}' = \frac{\mu_0}{4\pi}I\int\limits_{-L/2}^{L/2} \frac{1}{\sqrt{l^2+s^2}}\mathrm{d}l\boldsymbol{e}_z$$

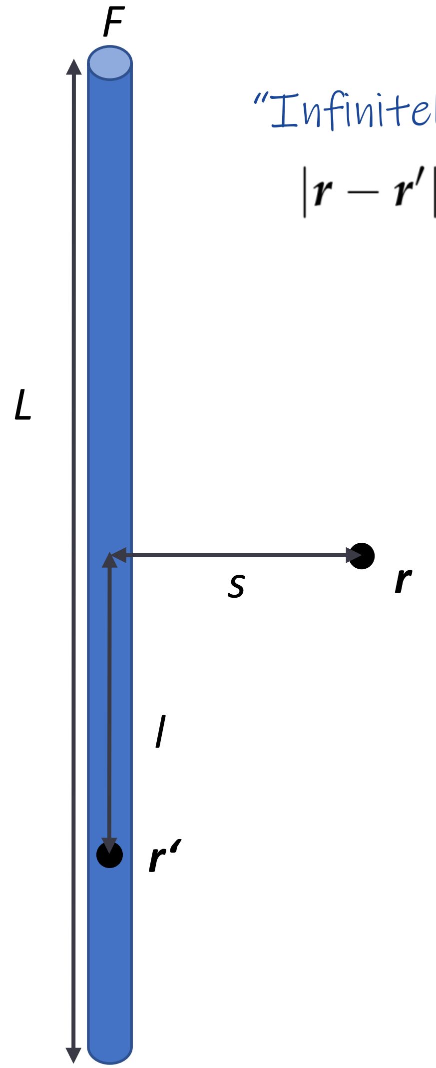
$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}I\ln\left[l+\sqrt{s^2+l^2}\right]_{l=-L/2}^{l=+L/2}\boldsymbol{e}_z$$

"Infinitely" long wire

$L \gg s$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{2\pi}I\ln\frac{L}{s}\boldsymbol{e}_z = \frac{\mu_0}{2\pi}I\ln\frac{L}{\sqrt{x^2+y^2}}\boldsymbol{e}_z$$

Charged wire



"Infinitely" thin wire

$$|\boldsymbol{r}-\boldsymbol{r}'| \gg \sqrt{F}$$

"Infinitely" long wire

$$L \gg s$$

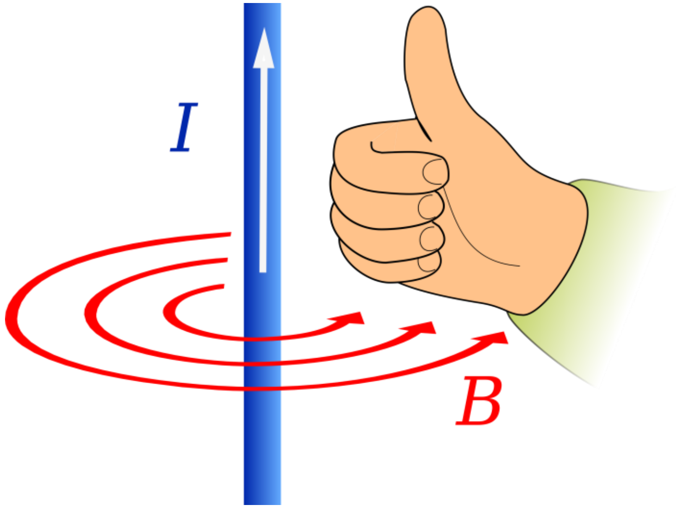
$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{2\pi} I \ln \frac{L}{s}\, \boldsymbol{e}_z = \frac{\mu_0}{2\pi} I \ln \frac{L}{\sqrt{x^2+y^2}}\, \boldsymbol{e}_z$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\boldsymbol{B} = \begin{pmatrix} \frac{\mu_0}{2\pi} I \frac{\sqrt{x^2+y^2}}{L} \frac{(-1/2)L}{\sqrt{x^2+y^2}^3} 2y \\ -\frac{\mu_0}{2\pi} I \frac{\sqrt{x^2+y^2}}{L} \frac{(-1/2)L}{\sqrt{x^2+y^2}^3} 2x \\ 0 \end{pmatrix}$$

$$\boldsymbol{B} = \frac{\mu_0}{2\pi} I \frac{1}{x^2+y^2} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

$$\boxed{\boldsymbol{B} = \frac{\mu_0}{2\pi} \frac{I}{s} \boldsymbol{e}_\varphi}$$



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Electrodynamics

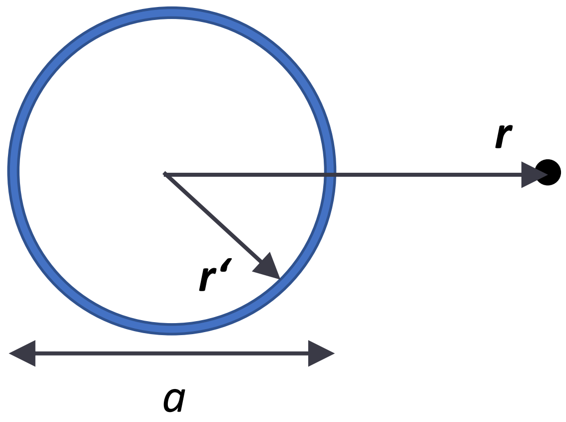
SECTION

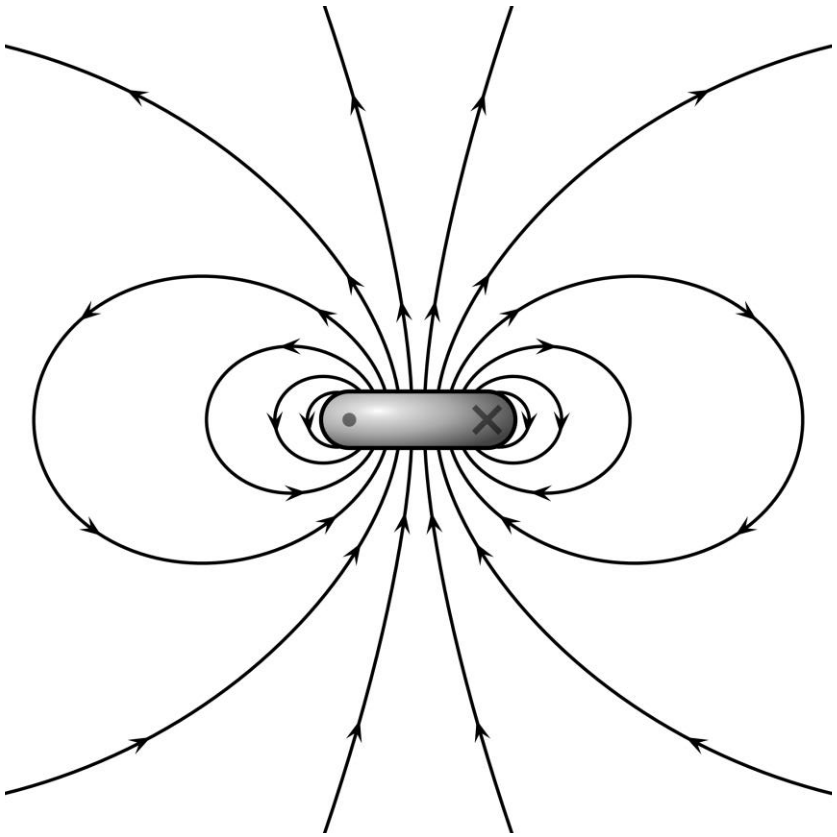
Magnetostatics

LECTURE

Current loop & magnetic dipole

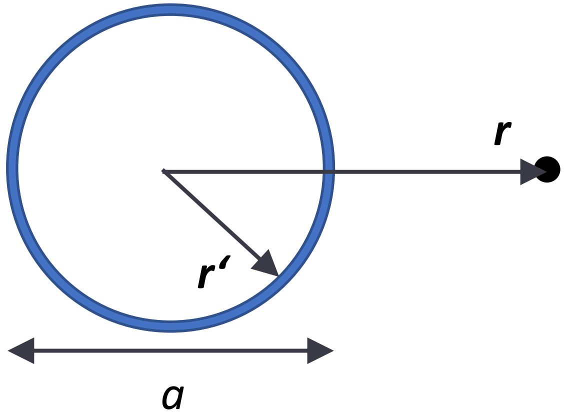
Current loop





https://upload.wikimedia.org/wikipedia/commons/5/56/VFPt_dipole_magnetic3.svg

Current loop



$$\boxed{\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'}$$

"Infinitely" thin wire

$|\boldsymbol{r}-\boldsymbol{r}'| \gg \sqrt{F}$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} I \oint_{\text{loop}} \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}\boldsymbol{r}'$$

- Truncated Taylor expansion

Far away

"Magnetic dipole"

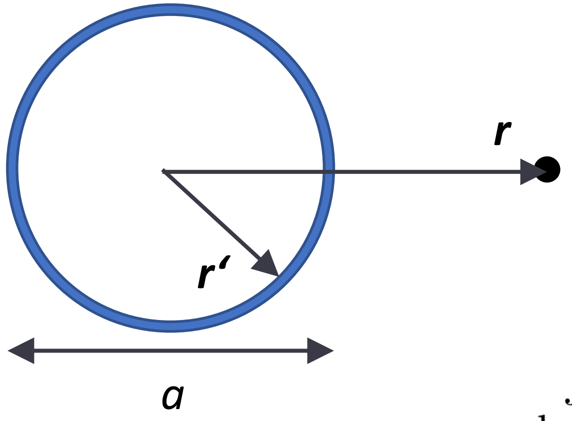
$|\boldsymbol{r}-\boldsymbol{r}'| \gg a$

$$\boldsymbol{A}(\boldsymbol{r}) \approx \frac{\mu_0}{4\pi} I \oint_{\text{loop}} \left[\frac{1}{r} - (\boldsymbol{r}'\cdot\nabla)\,\frac{1}{r}\right]\mathrm{d}\boldsymbol{r}'$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} I \frac{1}{r} \cancel{\oint_{\text{loop}} \mathrm{d}\boldsymbol{r}'} + \frac{\mu_0}{4\pi} I \frac{1}{r^3} \oint_{\text{loop}} (\boldsymbol{r}'\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}'$$

$$\nabla\frac{1}{r} = -\frac{\boldsymbol{r}}{r^3}$$

Magnetic dipole



$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} I \frac{1}{r^3} \oint_{\text{loop}} (\boldsymbol{r}' \cdot \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}'$$

$$\oint_{\text{loop}} (\boldsymbol{r}' \cdot \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}' = \frac{1}{2} \oint_{\text{loop}} \left[\mathrm{d}\boldsymbol{r}' (\boldsymbol{r} \cdot \boldsymbol{r}') - \boldsymbol{r}' (\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r}')\right] + \frac{1}{2} \oint_{\text{loop}} \left[\mathrm{d}\boldsymbol{r}' (\boldsymbol{r} \cdot \boldsymbol{r}') + \boldsymbol{r}' (\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r}')\right]$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$$

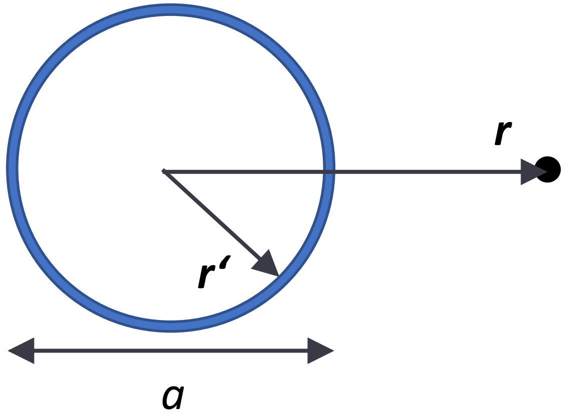
&

Product rule

$$\oint_{\text{loop}} (\boldsymbol{r}' \cdot \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}' = \frac{1}{2} \oint_{\text{loop}} \boldsymbol{r} \times (\mathrm{d}\boldsymbol{r}' \times \boldsymbol{r}') = \left[\frac{1}{2} \oint_{\text{loop}} (\boldsymbol{r}' \times \mathrm{d}\boldsymbol{r}')\right] \times \boldsymbol{r}$$

$$\cancel{+\frac{1}{2} \oint_{\text{loop}} \mathrm{d}\left[\boldsymbol{r}' (\boldsymbol{r} \cdot \boldsymbol{r}')\right]}$$

Magnetic dipole



$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} I \frac{1}{r^3} \oint_{\text{loop}} (\boldsymbol{r}' \cdot \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}'$$

$$\oint_{\text{loop}} (\boldsymbol{r}' \cdot \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}' = \left[\frac{1}{2} \oint_{\text{loop}} (\boldsymbol{r}' \times \mathrm{d}\boldsymbol{r}') \right] \times \boldsymbol{r}$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\frac{1}{2} I \oint_{\text{loop}} (\boldsymbol{r}' \times \mathrm{d}\boldsymbol{r}') \right] \times \boldsymbol{r}$$

Magnetic dipole moment

$$\boxed{\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \frac{\boldsymbol{m} \times \boldsymbol{r}}{r^3}}$$

$$\boxed{\boldsymbol{m} = \frac{1}{2} I \oint_{\text{loop}} (\boldsymbol{r}' \times \mathrm{d}\boldsymbol{r}')} = I \cdot A\boldsymbol{n}$$ ... for planar loops

Magnetic dipole

$$\boxed{\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{r}}{r^3}}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \nabla\times\boldsymbol{A}(\boldsymbol{r})$$

$$\nabla\times\frac{\boldsymbol{m}\times\boldsymbol{r}}{r^3} = \frac{1}{r^3}\nabla\times(\boldsymbol{m}\times\boldsymbol{r}) - (\boldsymbol{m}\times\boldsymbol{r})\times\nabla\frac{1}{r^3}$$

$$\nabla\times\varphi\boldsymbol{A} = \varphi\nabla\times\boldsymbol{A} - \boldsymbol{A}\times\nabla\varphi$$

$$= \frac{1}{r^3}\nabla\times(\boldsymbol{m}\times\boldsymbol{r}) + (\boldsymbol{m}\times\boldsymbol{r})\times\frac{3\boldsymbol{r}}{r^5}$$

$$\nabla\frac{1}{r^3} = -\frac{3\boldsymbol{r}}{r^5}$$

$$= \frac{1}{r^3}\left[\boldsymbol{m}(\nabla\cdot\boldsymbol{r}) - (\boldsymbol{m}\cdot\nabla)\boldsymbol{r}\right] + \frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{3\boldsymbol{r}\cdot\boldsymbol{r}}{r^5}\boldsymbol{m}$$

$$\nabla\times(\boldsymbol{A}\times\boldsymbol{B}) = (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} - \boldsymbol{B}(\nabla\cdot\boldsymbol{A}) + \boldsymbol{A}(\nabla\cdot\boldsymbol{B}) - (\boldsymbol{A}\cdot\nabla)\cdot\boldsymbol{B}$$

$$(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c} = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{b}\cdot\boldsymbol{c})\boldsymbol{a}$$

$$= \frac{3\boldsymbol{m}r^2}{r^5} - \frac{\boldsymbol{m}r^2}{r^5} + \frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{3\boldsymbol{m}r^2}{r^5}$$

$$\nabla\cdot\boldsymbol{r} = 3$$

$$(\boldsymbol{m}\cdot\nabla)\boldsymbol{r} = (m_x\frac{\partial}{\partial x} + m_y\frac{\partial}{\partial y} + m_z\frac{\partial}{\partial z})\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}m_x\\m_y\\m_z\end{pmatrix} = \boldsymbol{m}$$

$$\boxed{\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r} - \boldsymbol{m}r^2}{r^5}}$$

Magnetic dipole

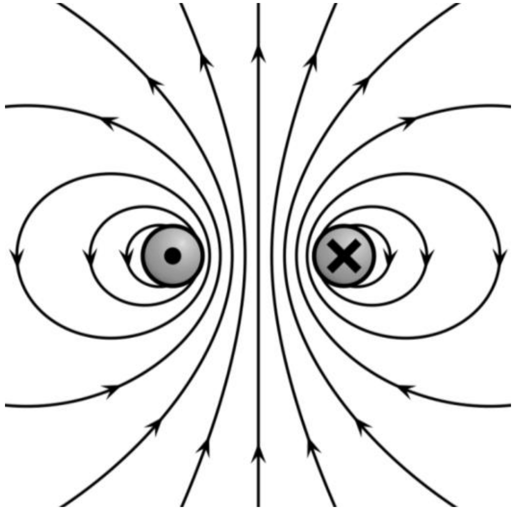
$$\boxed{\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r} - \boldsymbol{m}r^2}{r^5}}$$

$$\boxed{\boldsymbol{m} = \frac{1}{2}I\oint_{\text{loop}}(\boldsymbol{r}'\times \mathrm{d}\boldsymbol{r}')} = I\cdot A\boldsymbol{n}$$

Comparison: electric dipole

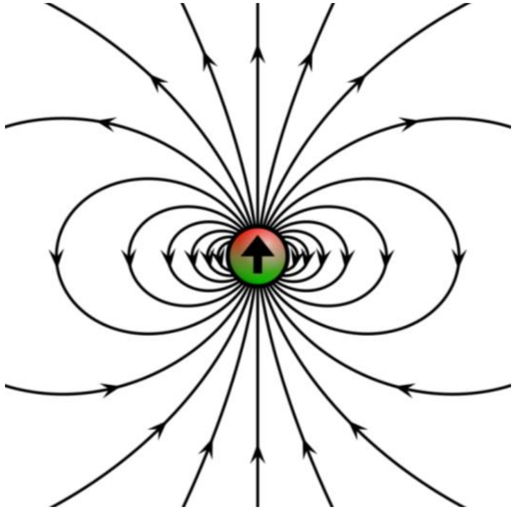
$$\boxed{\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{3(\boldsymbol{p}\cdot\boldsymbol{r})\boldsymbol{r} - \boldsymbol{p}r^2}{r^5}}$$

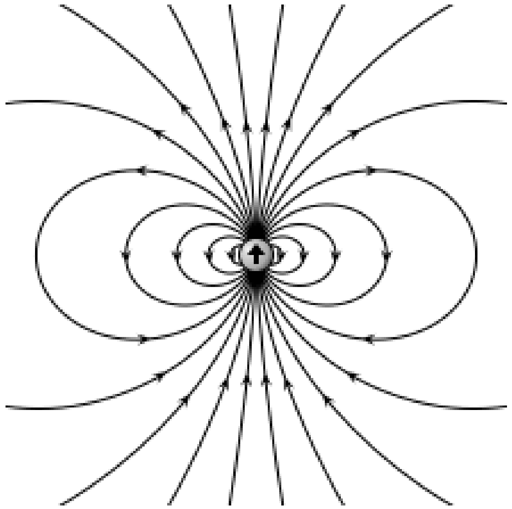
$$\boxed{\boldsymbol{p} = Q\boldsymbol{a}}$$



Zoom out

$|\boldsymbol{r}-\boldsymbol{r}'| \gg a$





https://upload.wikimedia.org/wikipedia/commons/7/76/VFPt_dipoles_magnetic.svg

https://upload.wikimedia.org/wikipedia/commons/a/aa/VFPt_dipole_animation_electric.gif

Electrodynamics

SECTION

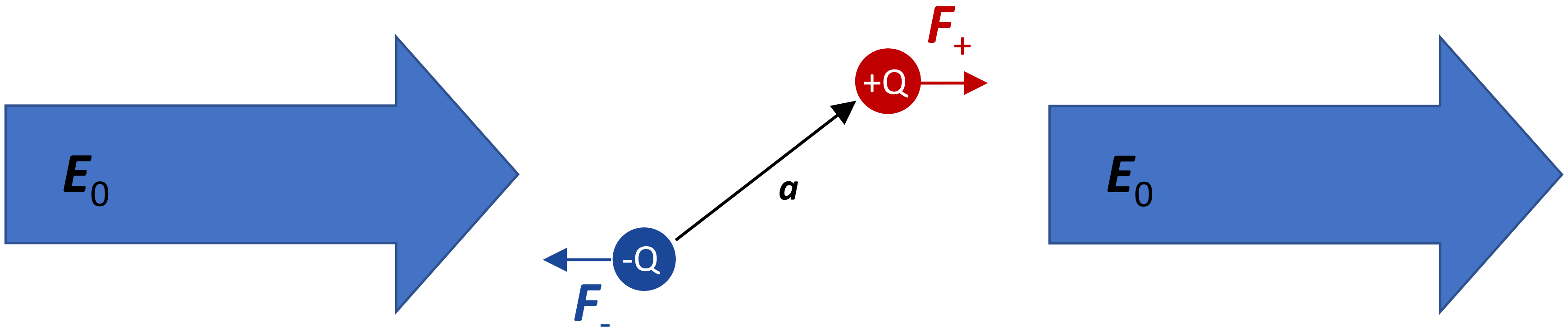
Magnetostatics

LECTURE

Force and torque acting on a magnetic dipole

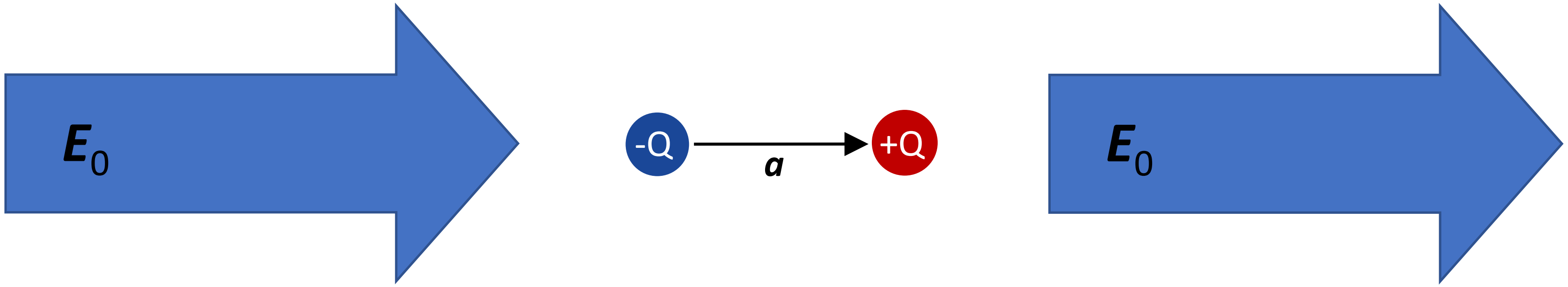
Electric dipole in a homogeneous *E* field

$$\boldsymbol{p} = Q\boldsymbol{a}$$



Electric dipole in a homogeneous *E* field

$$\boldsymbol{p} = Q\boldsymbol{a}$$

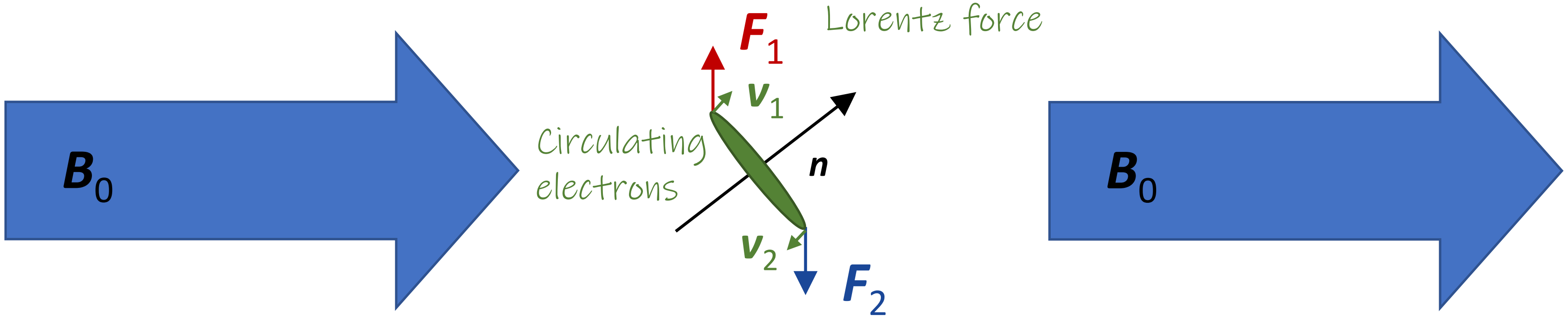


Reorientation $$\boldsymbol{M} = \boldsymbol{p} \times \boldsymbol{E}(\boldsymbol{r})$$

No motion $$\boldsymbol{F} = (\boldsymbol{p} \cdot \nabla)\boldsymbol{E}(\boldsymbol{r})$$

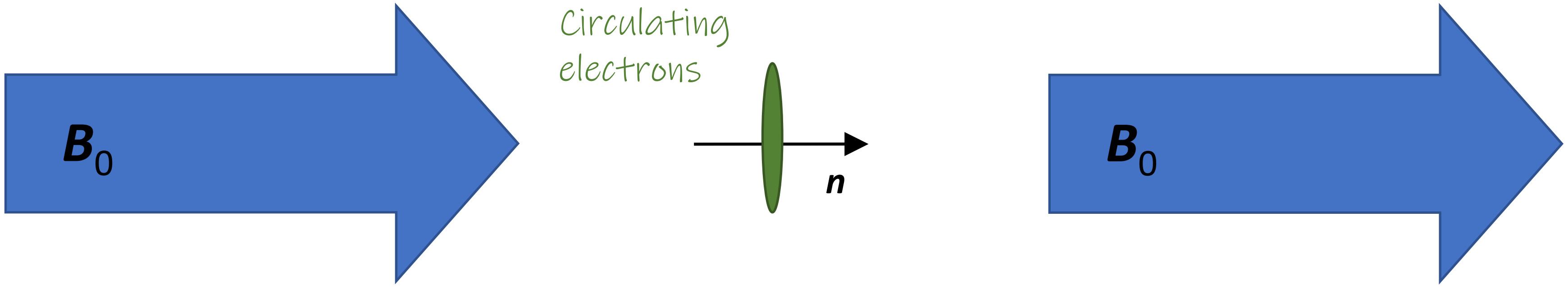
Magnetic dipole in a homogeneous *B* field

$$\boldsymbol{m} = \frac{1}{2} I \oint_{\text{loop}} (\boldsymbol{r}' \times \mathrm{d}\boldsymbol{r}') = I \cdot A\boldsymbol{n}$$



Magnetic dipole in a homogeneous *B* field

$$\boldsymbol{p} = Q\boldsymbol{a}$$



Electrostatics

Reorientation	$\boldsymbol{M} = \boldsymbol{m} \times \boldsymbol{B}(\boldsymbol{r})$	$\boldsymbol{M} = \boldsymbol{p} \times \boldsymbol{E}(\boldsymbol{r})$
No motion	$\boldsymbol{F} = (\boldsymbol{m} \cdot \nabla)\, \boldsymbol{B}(\boldsymbol{r})$	$\boldsymbol{F} = (\boldsymbol{p} \cdot \nabla) \boldsymbol{E}(\boldsymbol{r})$

Electrodynamics

SECTION

Magnetostatics

LECTURE

Energy in magnetostatics

Field energy

$$-\boldsymbol{j}\cdot\boldsymbol{E} = \frac{\partial}{\partial t}\left(\epsilon_0\frac{\boldsymbol{E}^2}{2} + \frac{1}{2\mu_0}\boldsymbol{B}^2\right) + \nabla\cdot\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0}$$

$$\boxed{\nu = \frac{\partial}{\partial t}w + \nabla\cdot\boldsymbol{S}}$$

Continuity equation for energy density

- Power density (energy generation density) $$\nu = -\boldsymbol{j}\cdot\boldsymbol{E}$$
- Energy density $$w = \epsilon_0\frac{\boldsymbol{E}^2}{2} + \frac{1}{2\mu_0}\boldsymbol{B}^2$$
- Poynting vector (energy-current density) $$\boldsymbol{S} = \frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0}$$

Magnetostatic energy

Recap: electrostatics

Now: magnetostatics

- Energy density

$$w = \epsilon_0 \frac{\boldsymbol{E}^2}{2} + \cancel{\frac{1}{2\mu_0}\boldsymbol{B}^2}$$

$$w = \cancel{\epsilon_0 \frac{\boldsymbol{E}^2}{2}} + \frac{1}{2\mu_0}\boldsymbol{B}^2$$

- Energy

$$\boxed{W_e = \int w_e \,\mathrm{d}V = \int \frac{\epsilon_0}{2}\boldsymbol{E}^2 \,\mathrm{d}V}$$

$$\boxed{W_e = \int w_e \,\mathrm{d}V = \int \frac{1}{2\mu_0}\boldsymbol{B}^2 \,\mathrm{d}V}$$

$$\boldsymbol{E}(\boldsymbol{r}) = -\nabla\varphi(\boldsymbol{r})$$

$$\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \frac{1}{\epsilon_0}\rho(\boldsymbol{r})$$

$$\nabla \times \boldsymbol{B}(\boldsymbol{r}) = \mu_0 \boldsymbol{j}(\boldsymbol{r})$$

no surface charges

$$\boxed{W_e = \frac{1}{2}\int \rho(\boldsymbol{r})\varphi(\boldsymbol{r}) \,\mathrm{d}V}$$

$$\boxed{W_e = \frac{1}{2}\int \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{A}(\boldsymbol{r}) \,\mathrm{d}V}$$

Electrodynamics

SECTION

Magnetostatics

LECTURE

Comparison to electrostatics

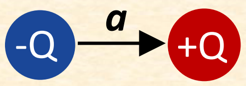
Electrostatics vs. Magnetostatics

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = 0$$
$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \frac{1}{\epsilon_0}\rho(\boldsymbol{r})$$

$$\boldsymbol{E}(\boldsymbol{r}) = -\nabla\varphi(\boldsymbol{r})$$
$$\Delta\varphi(\boldsymbol{r}) = -\frac{1}{\epsilon_0}\rho(\boldsymbol{r})$$

$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'$$
$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(\boldsymbol{r}')\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}V'$$

$$\boldsymbol{p} = Q\boldsymbol{a}$$



$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{3(\boldsymbol{p}\cdot\boldsymbol{r})\boldsymbol{r}-\boldsymbol{p}r^2}{r^5}$$
$$\boldsymbol{F} = (\boldsymbol{p}\cdot\nabla)\boldsymbol{E}(\boldsymbol{r}) \qquad \boldsymbol{M} = \boldsymbol{p}\times\boldsymbol{E}(\boldsymbol{r})$$

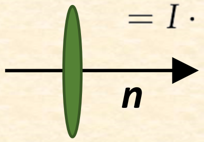
$$W_e = \int w_e\,\mathrm{d}V = \int \frac{\epsilon_0}{2}E^2\,\mathrm{d}V$$
$$W_e = \frac{1}{2}\int \rho(\boldsymbol{r})\varphi(\boldsymbol{r})\,\mathrm{d}V$$

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0$$
$$\nabla \times \boldsymbol{B}(\boldsymbol{r}) = \mu_0\boldsymbol{j}(\boldsymbol{r})$$

$$\boldsymbol{B}(\boldsymbol{r}) = \nabla\times\boldsymbol{A}(\boldsymbol{r})$$
$$\Delta\boldsymbol{A}(\boldsymbol{r}) = -\mu_0\boldsymbol{j}(\boldsymbol{r})$$
$$\nabla\cdot\boldsymbol{A}(\boldsymbol{r}) = 0$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int\frac{\boldsymbol{j}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}V'$$
$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \boldsymbol{j}(\boldsymbol{r}')\times\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}V'$$

$$\boldsymbol{m} = \frac{1}{2}I\oint(\boldsymbol{r}'\times\mathrm{d}\boldsymbol{r}') = I\cdot A\boldsymbol{n}$$



$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r}-\boldsymbol{m}r^2}{r^5}$$
$$\boldsymbol{F} = (\boldsymbol{m}\cdot\nabla)\boldsymbol{B}(\boldsymbol{r}) \qquad \boldsymbol{M} = \boldsymbol{m}\times\boldsymbol{B}(\boldsymbol{r})$$

$$W_e = \int w_e\,\mathrm{d}V = \int\frac{1}{2\mu_0}B^2\,\mathrm{d}V$$
$$W_e = \frac{1}{2}\int \boldsymbol{j}(\boldsymbol{r})\cdot\boldsymbol{A}(\boldsymbol{r})\,\mathrm{d}V$$