Electrodynamics

SECTION

Electromagnetic waves

LECTURE

Wave equation

Maxwell's equations in vaccuum

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

- Charges are sources of the electric field
- The magnetic field as no sources (no monopoles)
- Time-dependent magnetic fields generate electric fields
- Time-dependent electric fields and currents generate magnetic fields

Vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7} \, \frac{\mathrm{N}}{\mathrm{A}^2}$

Vacuum permittivity $\epsilon_0 = 8.854... \cdot 10^{-12} \, \frac{\mathrm{As}}{\mathrm{Vm}}$

Light as a wave

Maxwell's equations

$$\nabla \cdot \boldsymbol{E} = 0 \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \nabla \times \left(-\frac{\partial \boldsymbol{B}}{\partial t} \right) = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \nabla(\nabla \cdot \boldsymbol{E}) - \Delta \boldsymbol{E} \qquad \nabla \cdot \boldsymbol{E} = 0$$

Wave equation

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \Delta \boldsymbol{E} = 0$$

$$\mathbf{E}(\boldsymbol{r}, t) = \mathbf{E}_0 \, e^{i(\omega t - \mathbf{k} \cdot \boldsymbol{r})}$$

Light as an electromagnetic wave

Light as a wave

Maxwell's equations

$$\nabla \cdot \boldsymbol{E} = 0 \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\nabla \times (\nabla \times \boldsymbol{B}) = \mu_0 \epsilon_0 \nabla \times \frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \boldsymbol{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2}$$

$$\nabla \times (\nabla \times \boldsymbol{B}) = \nabla(\nabla \cdot \boldsymbol{B}) - \Delta \boldsymbol{B} = -\Delta \boldsymbol{B}$$

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} - \Delta \boldsymbol{B} = 0$$

Wave equation

$$\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0\, e^{i(\omega t - \boldsymbol{k} \cdot \boldsymbol{r})}$$

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Dispersion relation

Dispersion relation

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \Delta \boldsymbol{E} = 0$$

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$\left[\frac{(i\omega)^2}{c^2} - (-i\boldsymbol{k})^2\right] \boldsymbol{E}_0 e^{i(kx-\omega t)} = 0$$

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} - \Delta \boldsymbol{B} = 0$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{B}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$\left[\frac{(i\omega)^2}{c^2} - (-i\boldsymbol{k})^2\right] \boldsymbol{B}_0 e^{i(kx-\omega t)} = 0$$

$$\frac{(i\omega)^2}{c^2} - (-i\boldsymbol{k})^2 = 0$$

$$-\frac{\omega^2}{c^2} + k^2 = 0$$

$$\boxed{\omega = \pm ck}$$

Dispersion relation

Wave packets

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{B}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

Light as an electromagnetic wave

$$\omega = \pm ck$$

Dispersion relation

- All wave vectors *k* are allowed

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{\sqrt{2\pi}^3} \int \boldsymbol{f}(\boldsymbol{k}) e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega(\boldsymbol{k})t)}\, \mathrm{d}\boldsymbol{k}$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{1}{\sqrt{2\pi}^3} \int \tilde{\boldsymbol{f}}(\boldsymbol{k}) e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega(\boldsymbol{k})t)}\, \mathrm{d}\boldsymbol{k}$$

Fourier transform

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Orientation of k, B & E

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$i(\mathbf{E}_0 \cdot \mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = 0$$

$$\mathbf{E}_0 \cdot \mathbf{k} = 0$$

$$\mathbf{E}_0 \perp \mathbf{k}$$

Similar

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B}_0 \perp \mathbf{k}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$i(\mathbf{k} \times \mathbf{E}_0) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = i\omega \mathbf{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\frac{\mathbf{k}}{k} \times \mathbf{E} = \frac{\omega}{k} \mathbf{B}$$

$$\frac{\mathbf{k}}{k} \times \mathbf{E}_0 = c\mathbf{B}_0$$

$$\mathbf{k} \perp \mathbf{E} \perp \mathbf{B} \perp \mathbf{k}$$

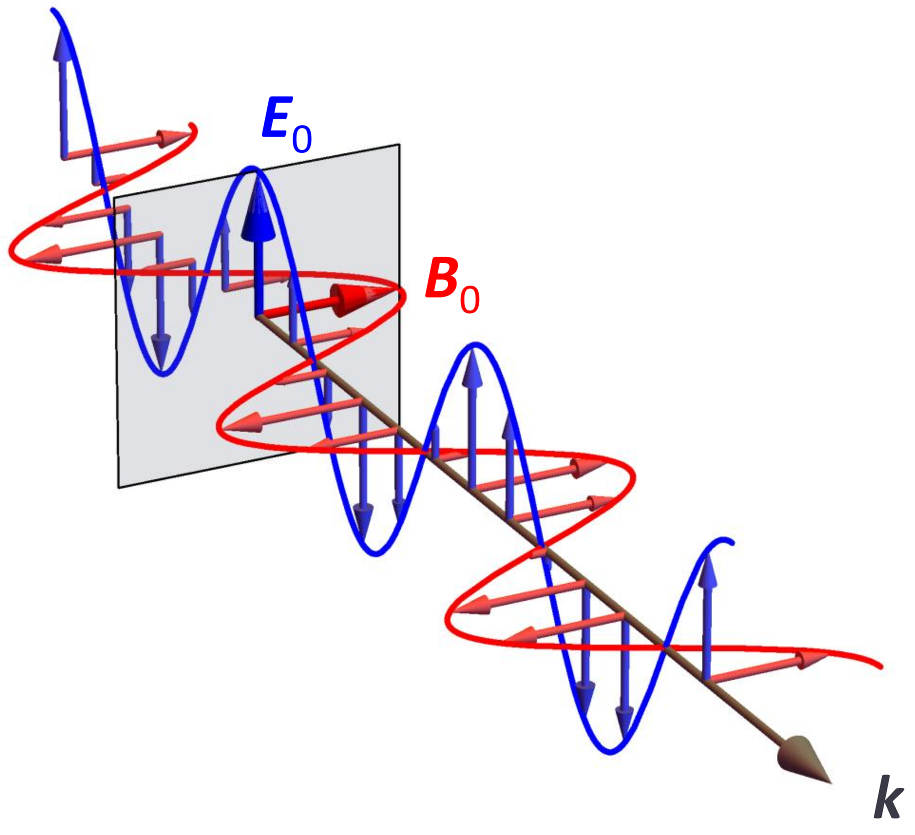
$$\mathbf{k} \perp \mathbf{E}_0 \perp \mathbf{B}_0 \perp \mathbf{k}$$

Orientation of *k*, *B* & *E*

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \qquad \boldsymbol{B} = \boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{k} \perp \boldsymbol{E} \perp \boldsymbol{B} \perp \boldsymbol{k}$$

$$\boldsymbol{k} \perp \boldsymbol{E}_0 \perp \boldsymbol{B}_0 \perp \boldsymbol{k}$$



Electrodynamics

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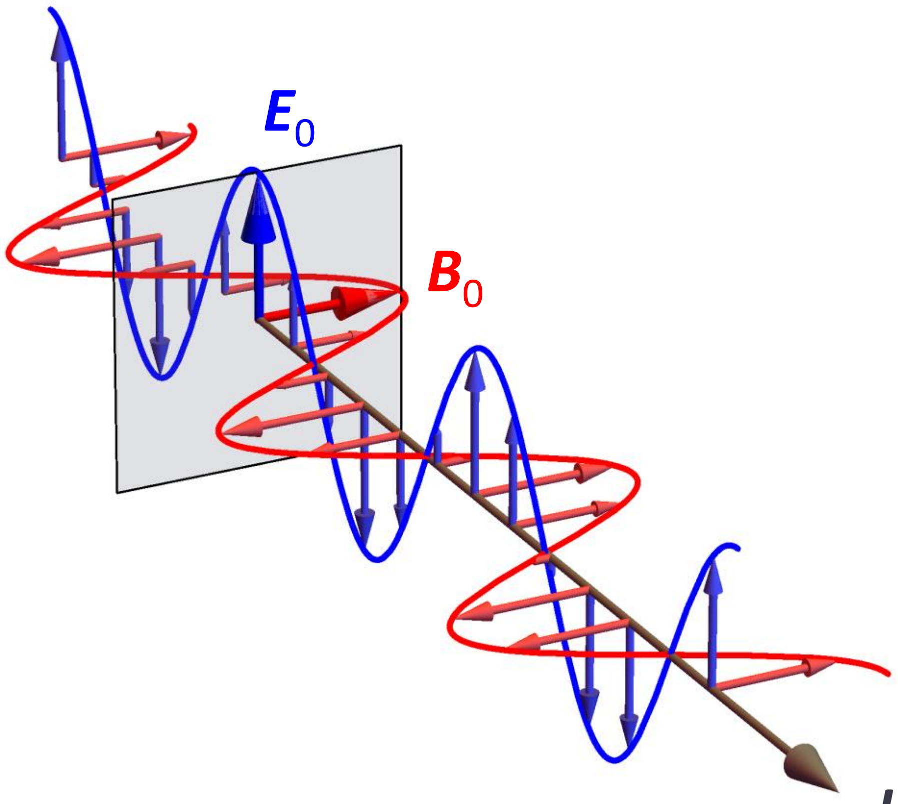
Polarization of light

Linear polarization

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \qquad \boldsymbol{B} = \boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{k} \perp \boldsymbol{E} \perp \boldsymbol{B} \perp \boldsymbol{k}$$
$$\boldsymbol{k} \perp \boldsymbol{E}_0 \perp \boldsymbol{B}_0 \perp \boldsymbol{k}$$

$$\boldsymbol{E}_0 = \boldsymbol{E}_R + i\boldsymbol{E}_i$$



- Special case $\boldsymbol{E}_r \parallel \boldsymbol{E}_i$

$$\mathrm{Re}\,(\boldsymbol{E}) = \boldsymbol{E}_r \left[\cos(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t) - \frac{E_i}{E_r}\sin(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)\right]$$

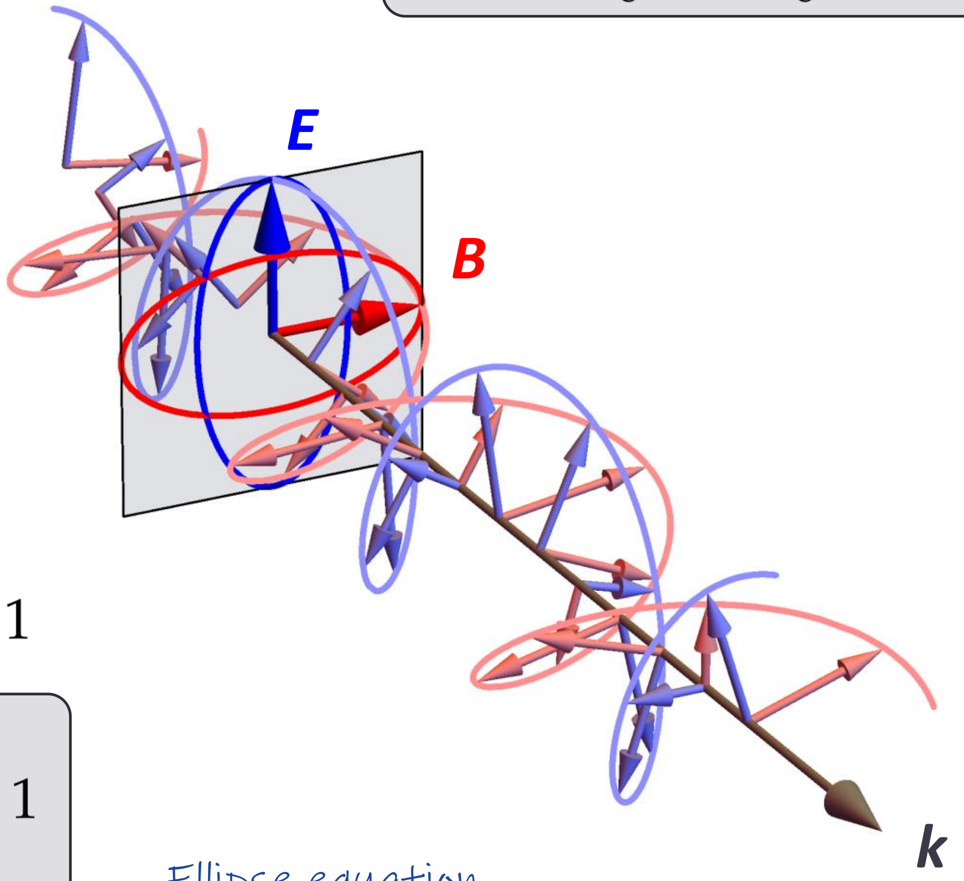
$$= \boldsymbol{E}_r \cos(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\delta)$$

Elliptical polarization

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{k} \perp \mathbf{E} \perp \mathbf{B} \perp \mathbf{k}$$
$$\mathbf{k} \perp \mathbf{E}_0 \perp \mathbf{B}_0 \perp \mathbf{k}$$

$$\mathbf{E}_0 = \mathbf{E}_R + i\mathbf{E}_i$$



- Special case $\mathbf{E}_r \perp \mathbf{E}_i$

$$\mathbf{k} \parallel \mathbf{z}$$
$$\mathbf{E}_r \parallel \mathbf{x}$$
$$\mathbf{E}_i \parallel \mathbf{y}$$

$$E_x = E_r \cos(\mathbf{k}\cdot\mathbf{r}-\omega t)$$
$$E_y = E_i \sin(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

$$\left(\frac{E_x}{E_r}\right)^2 = \cos^2(\mathbf{k}\cdot\mathbf{r}-\omega t)$$
$$\left(\frac{E_y}{E_i}\right)^2 = \sin^2(\mathbf{k}\cdot\mathbf{r}-\omega t)$$

$$\cos^2(a) + \sin^2(a) = 1$$

$$\left(\frac{E_x}{E_r}\right)^2 + \left(\frac{E_y}{E_i}\right)^2 = 1$$

Ellipse equation

Circular polarization

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} \qquad \boldsymbol{B} = \boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{k} \perp \boldsymbol{E} \perp \boldsymbol{B} \perp \boldsymbol{k}$$
$$\boldsymbol{k} \perp \boldsymbol{E}_0 \perp \boldsymbol{B}_0 \perp \boldsymbol{k}$$

$$\boldsymbol{E}_0 = \boldsymbol{E}_R + i\boldsymbol{E}_i$$

- Special case $\boldsymbol{E}_r \perp \boldsymbol{E}_i$

$$\left(\frac{E_x}{E_r}\right)^2 + \left(\frac{E_y}{E_i}\right)^2 = 1$$

$$E_r = \pm E_i$$