

$$T(n) = T(n-1) + 2n, T(0) = 0$$

i T(n)

$$1 \quad T(n) = \mathbf{T(n-1) + 2n}$$

$$T(n-1) = T(n-1-1) + 2(n-1)$$

$$= \mathbf{T(n-2) + 2n - 2}$$

$$2 \quad T(n) = \mathbf{T(n-2) + 2n - 2 + 2n} = \mathbf{T(n-2) + 2(2n) - 2}$$

$$T(n-2) = T(n-2-1) + 2(n-2)$$

$$= \mathbf{T(n-3) + 2n - 4}$$

$$3 \quad T(n) = \mathbf{T(n-3) + 2n - 4 + 2(2n) - 2} = \mathbf{T(n-3) + 3(2n) - 2 - 4}$$

$$T(n-3) = T(n-3-1) + 2(n-3)$$

$$= \mathbf{T(n-4) + 2n - 6}$$

$$4 \quad T(n) = \mathbf{T(n-4) + 2n - 6 + 3(2n) - 2 - 4} = \mathbf{T(n-4) + 4(2n) - 2 - 4 - 6}$$

(Look at all of the Bold portions to find a pattern for the "i-th" term)

$$\text{i-th Term: } T(n-i) + i(2n) - 2 \sum_{k=1}^{i-1} k$$

Relate the base case $T(0) = 0$ to "i-th" term: $T(n-i) = 0, n - i = 0, n = i$

Replace "i" with n in the "i-th" term formula and solve:

$$T(n) = T(n-n) + n(2n) - 2 \sum_{k=1}^{n-1} k$$

$$T(n) = T(0) + 2n^2 - 2 \sum_{k=1}^{n-1} k \quad 2 \sum_{k=1}^{n-1} k \text{ simplified to closed-form} = 2 \cdot \frac{(n-1)(n)}{2}$$

$$T(n) = 0 + 2n^2 - 2 \cdot \frac{(n-1)(n)}{2}$$

$$T(n) = 2n^2 - 2 \cdot \frac{(n-1)(n)}{2}$$

$$T(n) = 2n^2 - (n^2 - n)$$

$$T(n) = 2n^2 - n^2 + n$$

$$T(n) = \mathbf{n^2} + n$$

Consider term of largest magnitude: $\Theta(n^2)$