**Section 5: Finite Volume Methods for the Navier Stokes Equations**

In this initial lecture we introduce the two-dimensional Navier-Stokes Equations

continuity equation:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

x-momentum:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}+\lambda\nabla\cdot\bar{u}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]$$

y-momentum:

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}+\lambda\nabla\cdot\bar{u}\right)$$

The above equations are expressed in what is known as the non-conservative form. For implementation of the finite-volume method, the conservation-law form is appropriate. The equivalent equations in conservation law form appear as follows:

x-momentum

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}+\lambda\nabla\cdot\bar{u}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]$$

y-momentum

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}+\lambda\nabla\cdot\bar{u}\right)$$

How? Let's look at the unsteady and convection terms of the x-momentum equation:

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}\equiv$$

$$\rho\frac{\partial u}{\partial t}+u\frac{\partial\rho}{\partial t}+u\frac{\partial(\rho u)}{\partial x}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+u\frac{\partial(\rho v)}{\partial y}$$

Note the terms highlighted in red represent the continuity equation multiplied by $u$, hence sum to zero. Hence the two forms of expressing the left side of the momentum equations are equivalent.

The momentum equations may also be rearranged on the right side as:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)= -\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial x}(\lambda\nabla\cdot\bar{u})$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)= -\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\left[\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial y}(\lambda\nabla\cdot\bar{u})$$

For each equation, the viscous terms (in the square brackets) sum to zero for a constant property, incompressible flow, as does the last term in the equations.

How? Let's look at the terms from the x-momentum equation:

$$\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right)\right]\equiv\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial y}\right)\equiv\mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

Finally, denoting the bracketed viscous terms and the 2nd viscosity coefficient terms as Sx and Sy respectively, the momentum equations may be written in conservation-law form as:

x-momentum

$$\frac{\partial(\rho u)}{\partial t}+\nabla\cdot(\rho\bar{u}u)=-\frac{\partial p}{\partial x}+\nabla\cdot(\mu\nabla u)+Sx$$

y-momentum

$$\frac{\partial(\rho v)}{\partial t}+\nabla\cdot(\rho\bar{u}v)=-\frac{\partial p}{\partial y}+\nabla\cdot(\mu\nabla v)+Sy$$

For a constant property, incompressible flow the source terms are zero.

**Finite Volume Method**

To implement the finite volume method, we integrate this form of the governing equations over a control volume. By applying the Gauss divergence theorem we convert the convective and viscous volume integral terms to surface integrals. For the x-momentum equation:

$$\int [\nabla \cdot (\rho \bar{u} u)]\, dV = \int \rho u(\bar{u} \cdot \hat{n})\, dA$$

and

$$\int [\nabla \cdot (\mu \nabla u)] dV = \int (\mu \nabla u) \cdot \hat{n} dA$$

The pressure gradient term is evaluated as a volume integral.

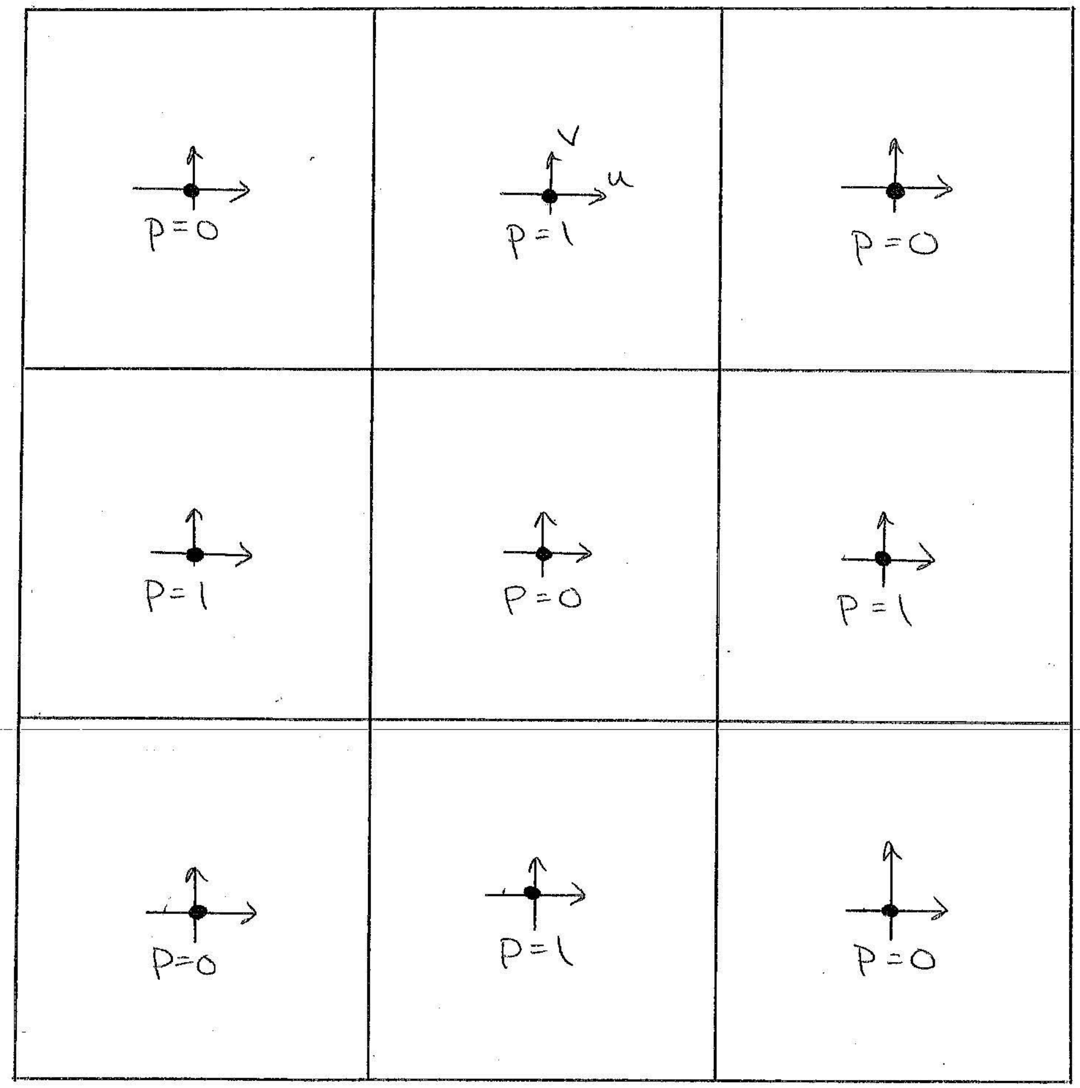
The x-momentum equation becomes:

$$\int \rho u(\bar{u} \cdot \hat{n})\, dA = -\int \frac{\partial p}{\partial x} dV + \int (\mu \nabla u) \cdot \hat{n} dA$$

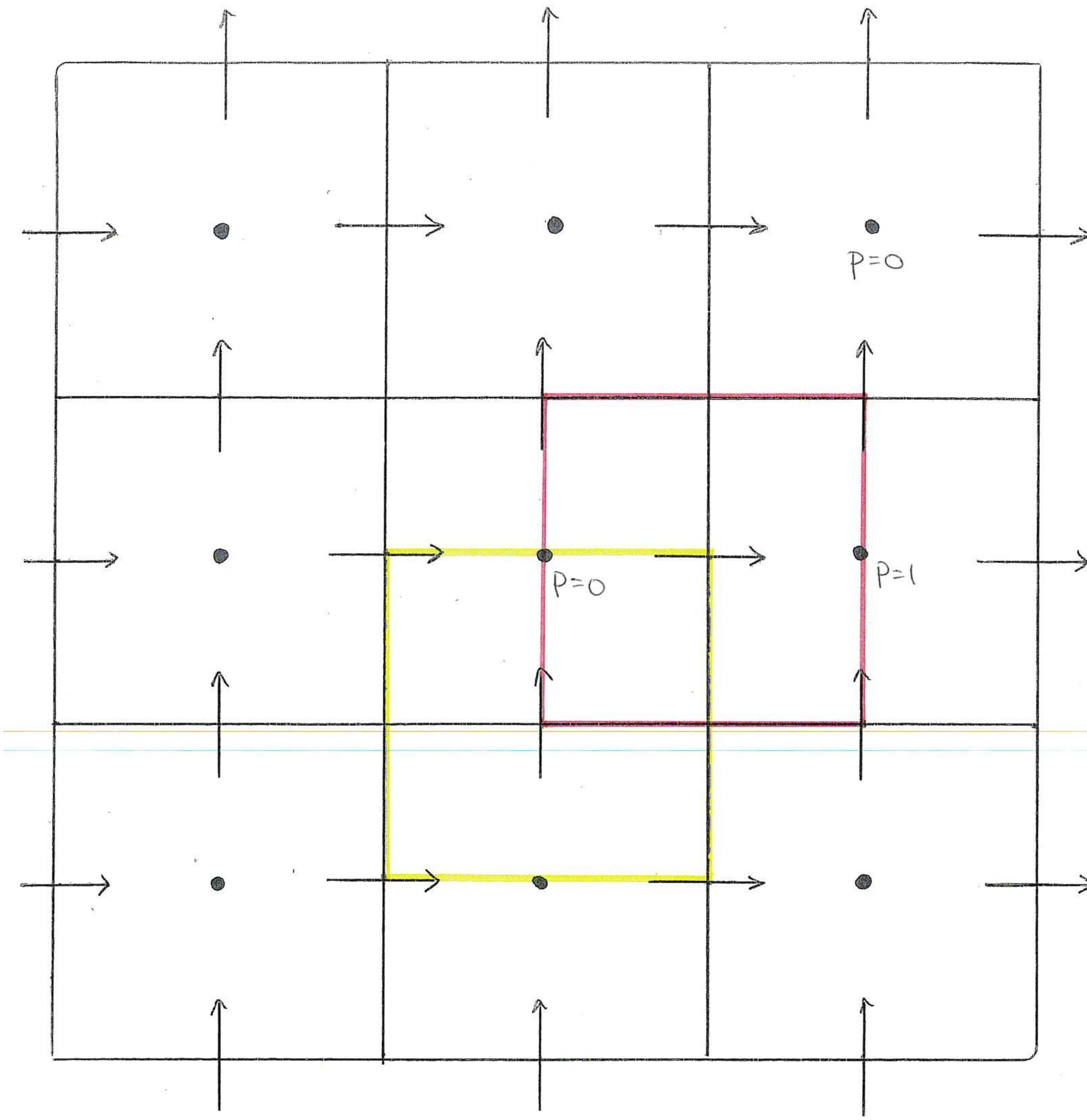
Similarly for the y-momentum equation:

$$\int \rho v(\bar{u} \cdot \hat{n})\, dA = -\int \frac{\partial p}{\partial y} dV + \int (\mu \nabla v) \cdot \hat{n} dA$$

In the following lectures we will look at the evaluation of these integrals over a control volume. However, our first step, discussed in the next lecture, is to look at the placement of the unknowns on the mesh.




collocated variable arrangement




Staggered variable arrangement

**The Semi-Implicit Method for Pressure-Linked Equation (SIMPLE) Procedure**

1) Guess values for u, v, p.

2) Solve the momentum equations for u and v. The resulting velocities will satisfy momentum, but not mass conservation.

3) Solve a pressure correction equation for a pressure correction p'.

4) Update the guessed pressure using the just-computed pressure correction, p'.

5) Update the velocities resulting from the solution to the momentum equations with velocity corrections, u' and v' (which are directly related to the computed pressure correction, p'). These new velocities will now satisfy mass conservation, but no longer satisfy momentum.

6) Go back to step 1 where the guessed values are now the results from steps 4 (for pressure) and 5 (for velocities).

The process continues until convergence to a pressure distribution that, when used in the momentum equations, results in both mass conserving and momentum conserving velocities out of the momentum solvers.

In the following lectures we will derive:

1) the discretized momentum equations

2) expressions for velocity corrections in terms of pressure corrections

3) the discretized continuity equation, which leads to the pressure correction equation

**Discretization of x-momentum equation**:

$$\int \rho u(\bar{u} \cdot \hat{n})\, dA = -\int \frac{\partial p}{\partial x} dV + \int (\mu \nabla u) \cdot \hat{n} dA$$

$$\rho u u|_w^e \Delta y + \rho v u|_s^n \Delta x = (P_w - P_e)\Delta y + \mu \frac{\partial u}{\partial x}\Big|_w^e \Delta y + \mu \frac{\partial u}{\partial y}\Big|_s^n \Delta x$$

Define:

$$\dot{m}_e = (\rho u)_e \Delta y \qquad \dot{m}_w = (\rho u)_w \Delta y$$

$$\dot{m}_n = (\rho v)_n \Delta x \qquad \dot{m}_s = (\rho v)_s \Delta x$$

Rewrite the discretized equation as:

$$\dot{m}_e u_e - \dot{m}_w u_w + \dot{m}_n u_n - \dot{m}_s u_s = (P_w - P_e)\Delta y + \mu_e \frac{U_E - U_P}{(\delta x)_e} \Delta y - \mu_w \frac{U_P - U_W}{(\delta x)_w} \Delta y + \mu_n \frac{U_N - U_P}{(\delta y)_n} \Delta x - \mu_s \frac{U_P - U_S}{(\delta y)_s} \Delta x$$

Group the coefficients of $U_P$:

$$\dot{m}_e u_e - \dot{m}_w u_w + \dot{m}_n u_n - \dot{m}_s u_s = (P_w - P_e)\Delta y - \left[\frac{\mu_e \Delta y}{(\delta x)_e} + \frac{\mu_w \Delta y}{(\delta x)_w} + \frac{\mu_n \Delta x}{(\delta y)_n} + \frac{\mu_s \Delta x}{(\delta y)_s}\right] U_P + \frac{\mu_e \Delta y}{(\delta x)_e} U_E + \frac{\mu_w \Delta y}{(\delta x)_w} U_W + \frac{\mu_n \Delta x}{(\delta y)_n} U_N + \frac{\mu_s \Delta x}{(\delta y)_s} U_S$$

Need to interpolate to find face values for $U_e, U_w, U_n, U_s$

We will use 1st order upwinding hence flow direction must be taken into account. We use the *max* function for that:

$$
\begin{aligned}
&max[\dot{m}_e, 0]U_P - max[-\dot{m}_e, 0]U_E + \\
&max[-\dot{m}_w, 0]U_P - max[\dot{m}_w, 0]U_W + \\
&max[\dot{m}_n, 0]U_P - max[-\dot{m}_n, 0]U_N + \\
&max[-\dot{m}_s, 0]U_P - max[\dot{m}_s, 0]U_S = \\
&\qquad (P_w - P_e)\Delta y - \left[\frac{\mu_e \Delta y}{(\delta x)_e} + \frac{\mu_w \Delta y}{(\delta x)_w} + \frac{\mu_n \Delta x}{(\delta y)_n} + \frac{\mu_s \Delta x}{(\delta y)_s}\right] U_P + \\
&\qquad \frac{\mu_e \Delta y}{(\delta x)_e} U_E + \frac{\mu_w \Delta y}{(\delta x)_w} U_W + \frac{\mu_n \Delta x}{(\delta y)_n} U_N + \frac{\mu_s \Delta x}{(\delta y)_s} U_S
\end{aligned}
$$

Recall we need to ensure that

$$A_P = A_E + A_W + A_N + A_S + (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s)$$

It currently doesn't. Note the difference in the signs within the *max* functions.

We can rewrite the terms multiplying $U_P$ as:

$$\begin{Bmatrix} max[\dot{m}_e, 0] + max[-\dot{m}_w, 0] + \\ max[\dot{m}_n, 0] + max[-\dot{m}_s, 0] \end{Bmatrix} U_P \equiv$$

$$\begin{Bmatrix} max[-\dot{m}_e, 0] + max[\dot{m}_w, 0] + \\ max[-\dot{m}_n, 0] + max[\dot{m}_s, 0] + \\ (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s) \end{Bmatrix} U_P$$

To check this, let's assume a 1D flow west-to-east so that $\dot{m}_e$ and $\dot{m}_w$ are both positive:

$$\dot{m}_e U_P = \dot{m}_w U_P + (\dot{m}_e - \dot{m}_w) U_P$$

or

$$\dot{m}_e U_P = \dot{m}_e U_P$$

If we assume flow from east-to-west so that $\dot{m}_e$ and $\dot{m}_w$ are both negative:

$$\dot{m}_w U_P = \dot{m}_e U_P + (-\dot{m}_e + \dot{m}_w) U_P$$

or

$$\dot{m}_w U_P = \dot{m}_w U_P$$

Now, we can define our coefficients as:

$$A_E^u = max[-\dot{m}_e, 0] + \frac{\mu_e \Delta y}{(\delta x)_e}$$

$$A_W^u = max[\dot{m}_w, 0] + \frac{\mu_w \Delta y}{(\delta x)_w}$$

$$A_N^u = max[-\dot{m}_n, 0] + \frac{\mu_n \Delta x}{(\delta y)_n}$$

$$A_S^u = max[\dot{m}_s, 0] + \frac{\mu_s \Delta x}{(\delta y)_s}$$

So that:

$$A_P^u = A_E^u + A_W^u + A_N^u + A_S^u + (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s)$$

The discretized x-momentum equation is then written as:

$$A_P^u U_P = A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y$$

(Note that if you sufficiently converge your pressure corrections (see pg. 13-15) such that the corrected velocities conserve mass over each cell, then the $(\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s)$ term in the definition of $A_P^u$ will be very close to zero. In that case, I have found that the code will converge even if those terms are neglected.)

## Solution Procedure

Next, we will build an under-relaxation procedure into the iterative solver, since the equation is nonlinear.

$$U_P = \frac{1}{A_P^u}\left(A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y\right)$$

Add and subtract $U_P^{old}$ from the right side:

$$U_P = U_P^{old} + \frac{\Omega}{A_P^u}\left[\left(A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y\right) - A_P^u U_P^{old}\right]$$

The above is an SOR type scheme. Let's keep working on it.

Multiply through by $A_P^u/\Omega$.

$$\frac{A_P^u}{\Omega} U_P = \frac{A_P^u}{\Omega} U_P^{old} + \left[A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y\right] - A_P^u U_P^{old}$$

or, rearranging:

$$\frac{A_P^u}{\Omega} U_P = (1 - \Omega)\frac{A_P^u}{\Omega} U_P^{old} + \left[A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y\right]$$

Now, redefine $\widetilde{A_P^u} = A_P^u/\Omega$ so that:

$$U_P = (1 - \Omega)U_P^{old} + \frac{1}{\widetilde{A_P^u}}\left[A_E^u U_E + A_W^u U_W + A_N^u U_N + A_S^u U_S + (P_w - P_e)\Delta y\right]$$

where, to under relax the solution, $\Omega < 1$.

Following the same procedure, the discretized y-momentum equation becomes (where $\widetilde{A_P^v} = A_P^v/\Omega$):

$$V_P = (1-\Omega)V_P^{old} + \frac{1}{\widetilde{A_P^v}}[A_E^v V_E + A_W^v V_W + A_N^v V_N + A_S^v V_S + (P_s - P_n)\Delta x]$$

where:

$$A_E^v = max[-\dot{m}_e, 0] + \frac{\mu_e \Delta y}{(\delta x)_e}$$

$$A_W^v = max[\dot{m}_w, 0] + \frac{\mu_w \Delta y}{(\delta x)_w}$$

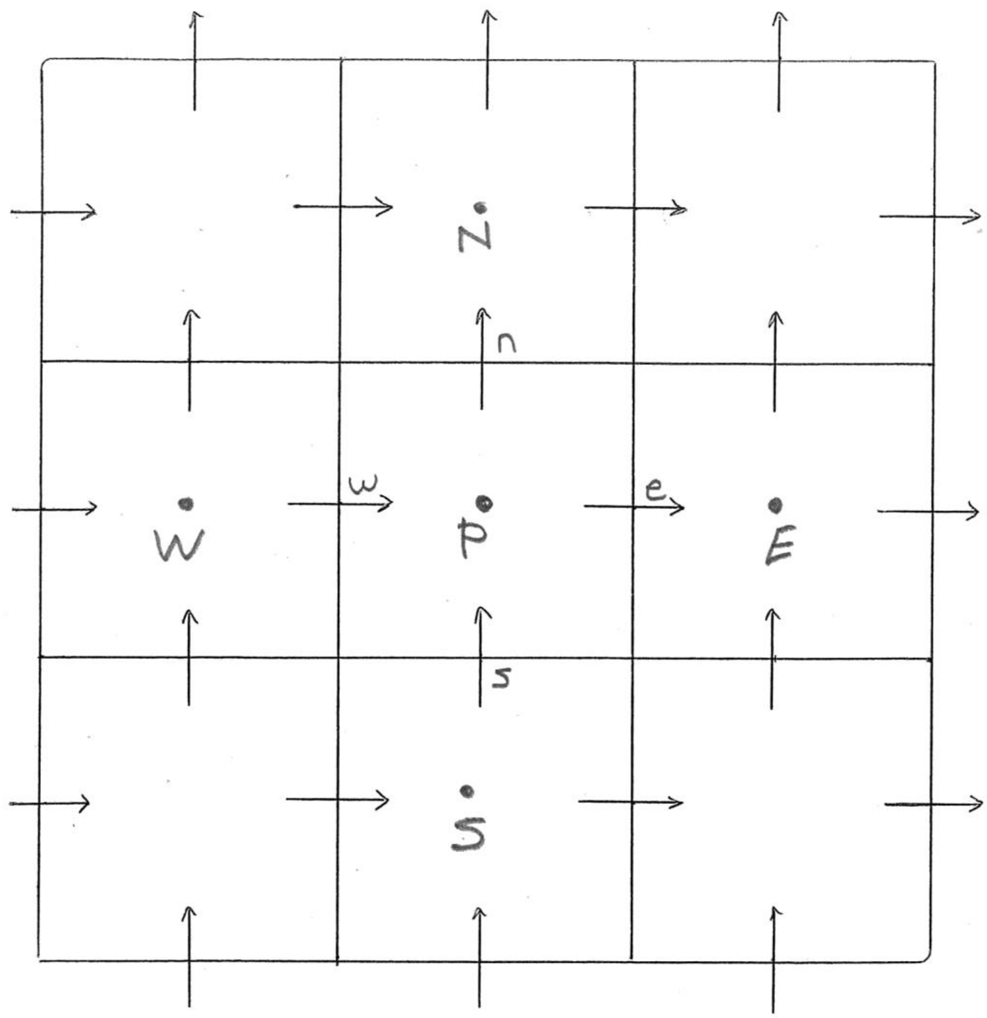
$$A_N^v = max[-\dot{m}_n, 0] + \frac{\mu_n \Delta x}{(\delta y)_n}$$

$$A_S^v = max[\dot{m}_s, 0] + \frac{\mu_s \Delta x}{(\delta y)_s}$$

So that:

$$A_P^v = A_E^v + A_W^v + A_N^v + A_S^v + (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s)$$

## Pressure Correction Procedure




$$A_{Pw}^{u} U_{w} = \sum_{nb} A_{nb}^{u} U_{nb} + (P_{W} - P_{P})\Delta y$$

$$A_{Pe}^{u} U_{e} = \sum_{nb} A_{nb}^{u} U_{nb} + (P_{P} - P_{E})\Delta y$$

$$A_{Ps}^{v} V_{s} = \sum_{nb} A_{nb}^{v} V_{nb} + (P_{S} - P_{P})\Delta x$$

$$A_{Pn}^{v} V_{n} = \sum_{nb} A_{nb}^{v} V_{nb} + (P_{P} - P_{N})\Delta x$$

Let * denote a guessed value, then write:

$$A_{Pw}^{u}U_{w}^{*} = \sum_{nb} A_{nb}^{u}\, U_{nb}^{*} + (P_{W}^{*} - P_{P}^{*})\Delta y$$

$$A_{Pe}^{u}U_{e}^{*} = \sum_{nb} A_{nb}^{u}\, U_{nb}^{*} + (P_{P}^{*} - P_{E}^{*})\Delta y$$

$$A_{Pn}^{v}V_{n}^{*} = \sum_{nb} A_{nb}^{v}\, V_{nb}^{*} + (P_{P}^{*} - P_{N}^{*})\Delta x$$

$$A_{Ps}^{v}V_{s}^{*} = \sum_{nb} A_{nb}^{v}\, V_{nb}^{*} + (P_{S}^{*} - P_{P}^{*})\Delta x$$

Now define:

$U = U^{*} + U'$

$V = V^{*} + V'$

$P = P^{*} + P'$

(i.e., U= guessed value + correction)

Subtract the second set of equations from the first:

$$A_{Pw}^{u}U_{w}' = \sum_{nb} A_{nb}^{u}\, U_{nb}' + (P_{W}' - P_{P}')\Delta y$$

etc.

This is an equation for the **corrections** to velocity and pressure.

If we neglect the summation term:

$$U'_w = \frac{\Delta y}{\widetilde{A^u_{Pw}}}(P'_W - P'_P)$$

$$U'_e = \frac{\Delta y}{\widetilde{A^u_{Pe}}}(P'_P - P'_E)$$

$$V'_s = \frac{\Delta x}{\widetilde{A^v_{Ps}}}(P'_S - P'_P)$$

$$V'_n = \frac{\Delta x}{\widetilde{A^v_{Pn}}}(P'_P - P'_N)$$

These are known as the velocity correction equations.

Note that all the $A^u_{Pw}$, etc. for the corrections need to be the tilde values in the computer code, i.e., $\widetilde{A^u_{Pw}}$, our iteration equation use the tilde value of the Ap's.

**Next, let's look at the (steady) continuity equation:**

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Discretize using the finite volume approach:

$$[(\rho U)_e - (\rho U)_w]\Delta y + [(\rho V)_n - (\rho V)_s]\Delta x = 0$$

Substitute:

$$U = U^* + U'$$

$$V = V^* + V'$$

$$P = P^* + P'$$

$$(\rho U_e^* + \rho U_e' - \rho U_w^* - \rho U_w')\Delta y + (\rho V_n^* + \rho V_n' - \rho V_s^* - \rho V_s')\Delta x = 0$$

Gather the "starred" and "prime" terms:

$$(\rho U_e' - \rho U_w')\Delta y + (\rho V_n' - \rho V_s')\Delta x + [(\rho U_e^* - \rho U_w^*)\Delta y + (\rho V_n^* - \rho V_s^*)\Delta x] = 0$$

The portion in yellow represents a "mass source term," *S*.

Now substitute velocity corrections:

$$\frac{\rho \Delta y^2}{\widetilde{A_{Pe}^u}}(P_P' - P_E') - \frac{\rho \Delta y^2}{\widetilde{A_{Pw}^u}}(P_W' - P_P') + \frac{\rho \Delta x^2}{\widetilde{A_{Pn}^v}}(P_P' - P_N') - \frac{\rho \Delta x^2}{\widetilde{A_{Ps}^v}}(P_S' - P_P') + S = 0$$

Gather the $P_P'$ terms on the left side, the remainder on the right:

$$\left(\frac{\rho\Delta y^2}{\widetilde{A^u_{Pe}}}+\frac{\rho\Delta y^2}{\widetilde{A^u_{Pw}}}+\frac{\rho\Delta x^2}{\widetilde{A^v_{Pn}}}+\frac{\rho\Delta x^2}{\widetilde{A^v_{Ps}}}\right)P'_P = \frac{\rho\Delta y^2}{\widetilde{A^u_{Pe}}}P'_E+\frac{\rho\Delta y^2}{\widetilde{A^u_{Pw}}}P'_W+\frac{\rho\Delta x^2}{\widetilde{A^v_{Pn}}}P'_N+\frac{\rho\Delta x^2}{\widetilde{A^v_{Ps}}}P'_S - S = 0$$

or:

$$A_P P'_P = A_E P'_E + A_W P'_W + A_N P'_N + A_S P'_S - S$$

Now, putting into the form of an SOR scheme:

$$P'_P = P'_P + \frac{\omega}{A_P}(A_E P'_E + A_W P'_W + A_N P'_N + A_S P'_S - S - A_P P'_P)$$

This is our equation over which we will iterate to find the pressure corrections.

**What about boundary conditions?**

We do not change velocities on boundaries using velocity correction terms. Hence, velocities normal to the boundaries must be set so that mass is conserved globally.

That is, on the east/west boundaries:

$U^* = U$ and $U' = 0$

This means that, for instance, on the east boundary:

$$U'_e = \frac{\Delta y}{A^e_{Pu}}(P'_P - P'_E) \equiv 0$$

Consequently, $P'_P = P'_E$

We can force this in our code by setting the coefficient $A_E$ to zero along the entire east boundary, or equivalently by setting $A^e_{Pu}$ to a very large number $(i.e., 10^{30})$. Similarly for the other boundaries.

Note that the pressure update must be under relaxed:

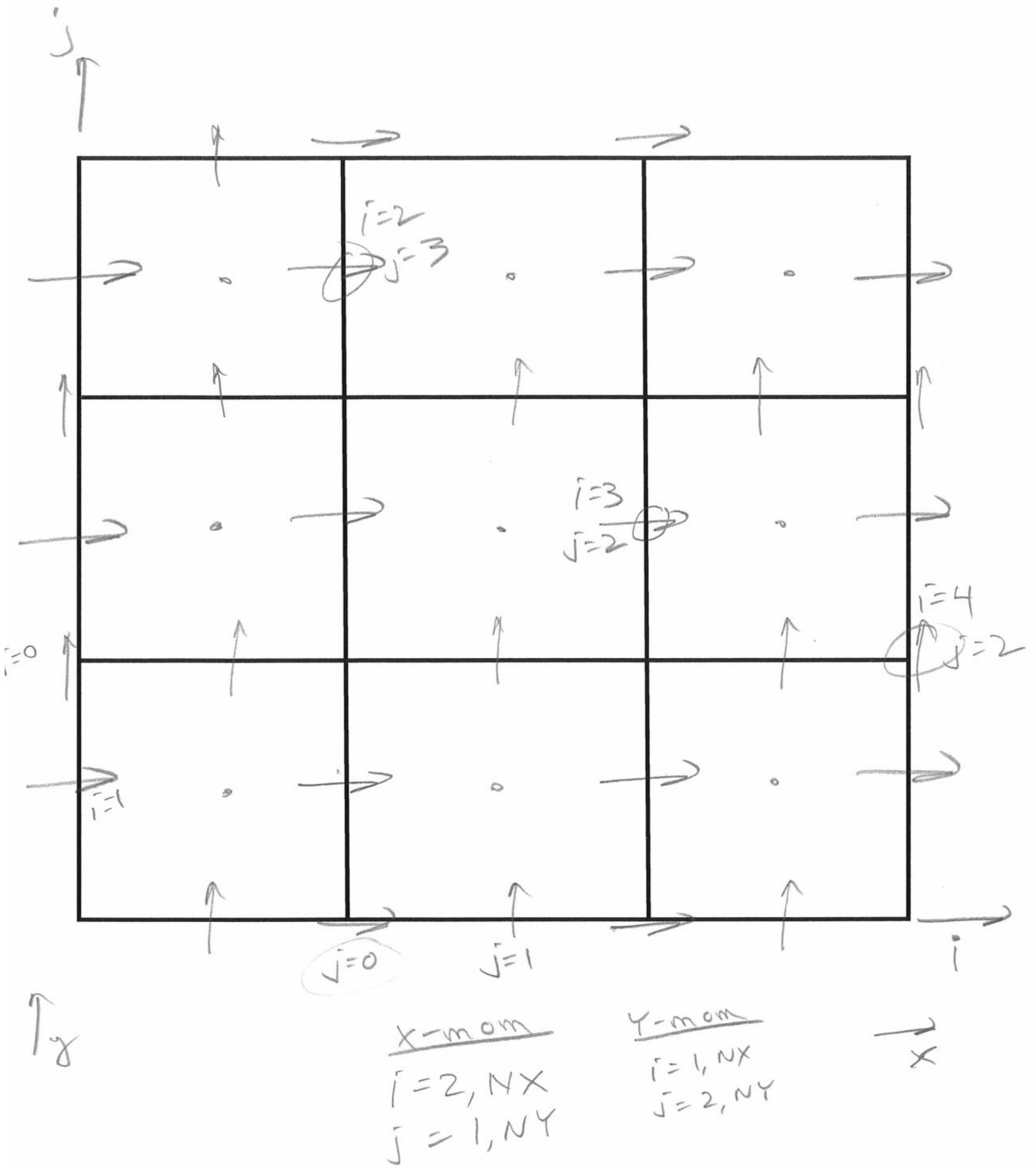
$P^{new} = P^{old} + \alpha P'$ where $0 < \alpha < 1$

Velocities are also corrected using the velocity correction equation. These corrections are not under relaxed as they conserve mass. The corrections look like:

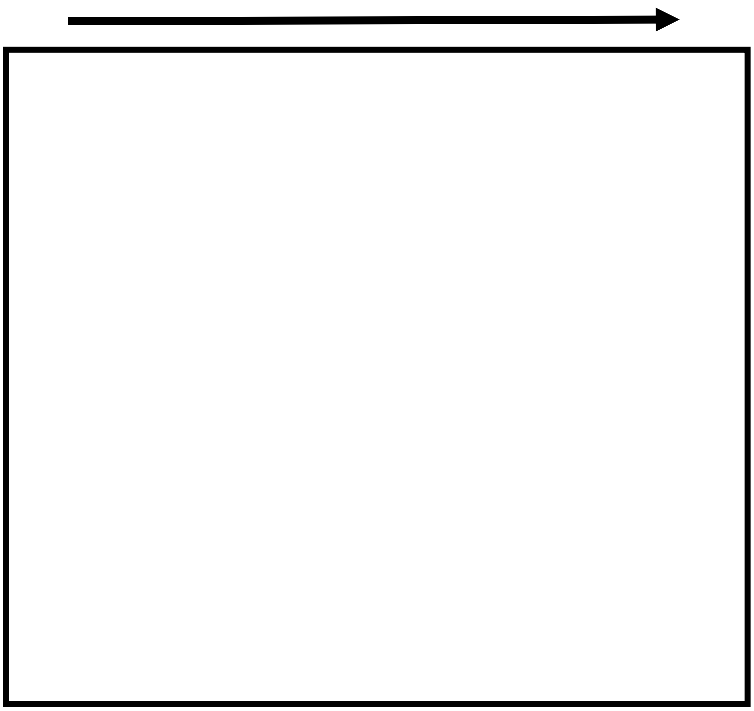
$$U_e^{new} = U_e^{old} + U'_e = U_e^{old} + \frac{\Delta y}{\widetilde{A^u_{Pe}}}(P'_P - P'_E)$$

Note the pressure correction equation itself is linear, so it can be over relaxed, that is, $1 < \omega < 2$

Suggested Index Notation
(and what is used in Fortran code)



The Driven Cavity Problem



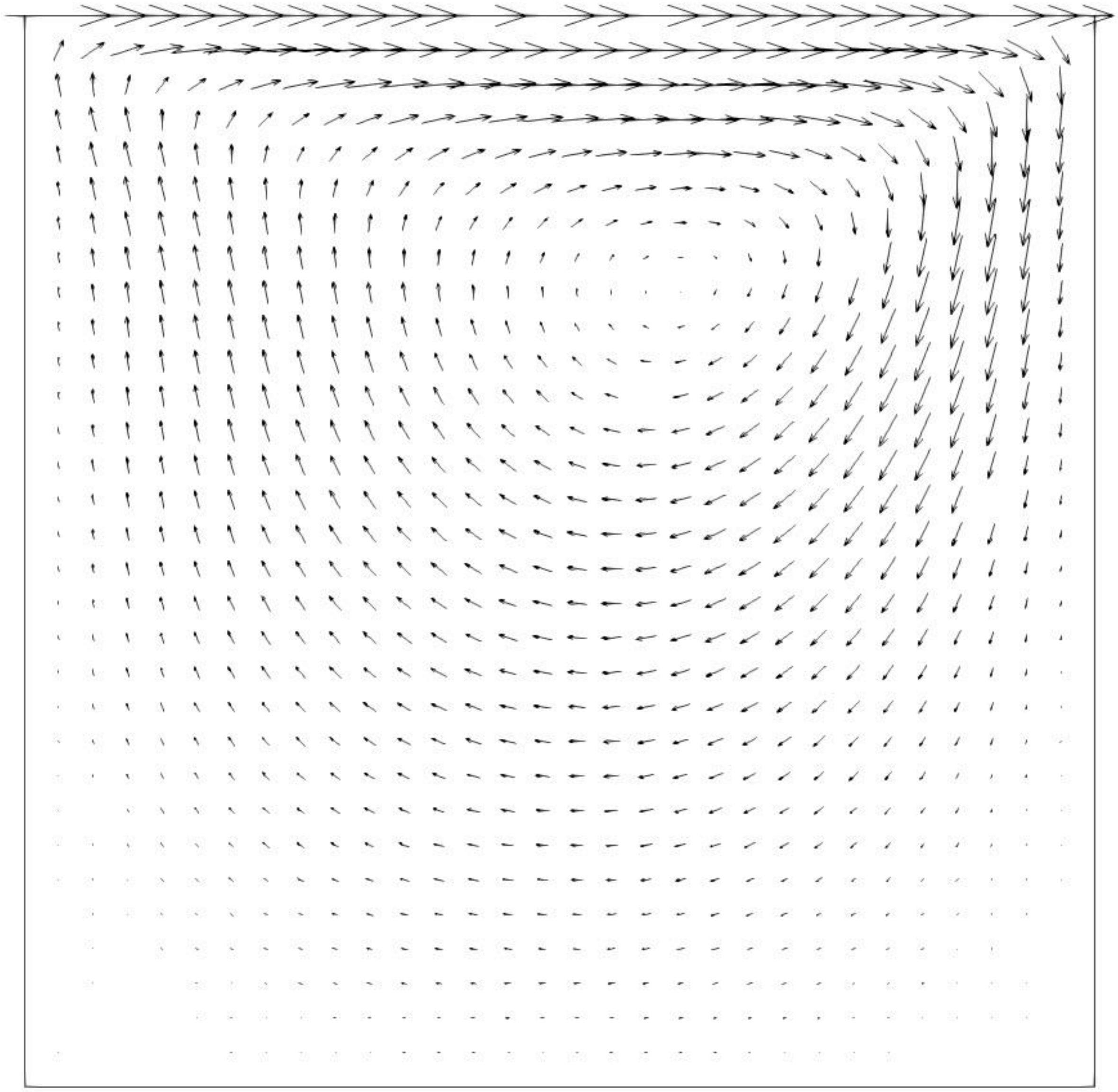
0<=x<=1

0<=y<=1

Let density = 1 kg/m^3; dynamic viscosity = 0.01 kg/m s.

North side moves left-to-right at 1 m/s.

Results in a Reynolds number of 100.



Developing Flow in a Channel

0<=x<=10
0<=y<=1

Let density = 1 kg/m^3; dynamic viscosity = 0.01 kg/m s.

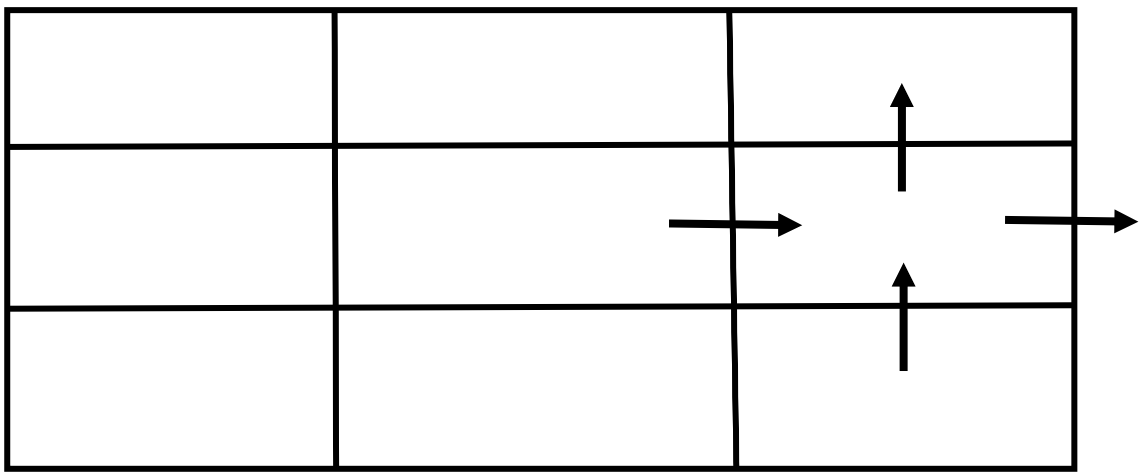
Inlet velocity =1 m/s.

Zero derivative velocity specification at outlet.

Reynolds number = 100.

**Other Boundary Conditions**

Pressure Boundary Condition

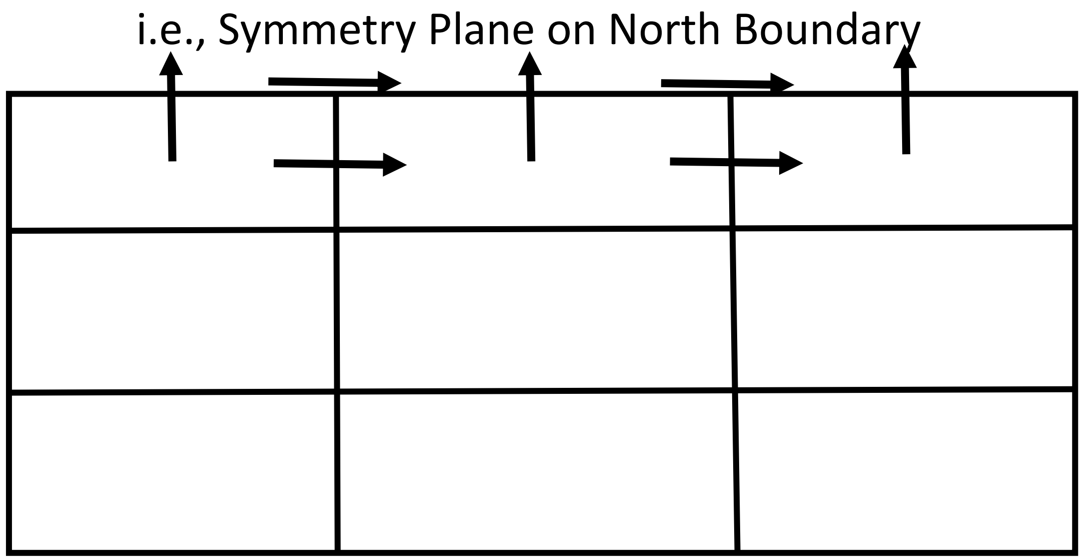


1) Set Ap in pressure correction equation to 10^30 on pressure boundary condition cells. This will force $P'$ to remain at 0, hence pressure will not be altered from initial specification.

2) Then, say on east end of domain, compute $U_e$ by enforcing continuity:

$$U_e = \frac{-(\rho VA)_n + (\rho VA)_s + (\rho UA)_w}{(\rho A)_e}$$

Symmetry Boundary Condition




1) Set V=0 along north face of top wall cells (as a boundary condition).

2) Set du/dy = 0 along north face of top wall cells.

3) In pressure correction routine, set the $A_N$ coefficient along top wall cells to 0.

Procedure to Generate Fully Developed Flow

Velocity at inlet plane updated to velocity at outlet plane as the solver proceeds through the outer iterations.
Useful for creating a fully developed flow profile for input into another simulation as inlet boundary condition.

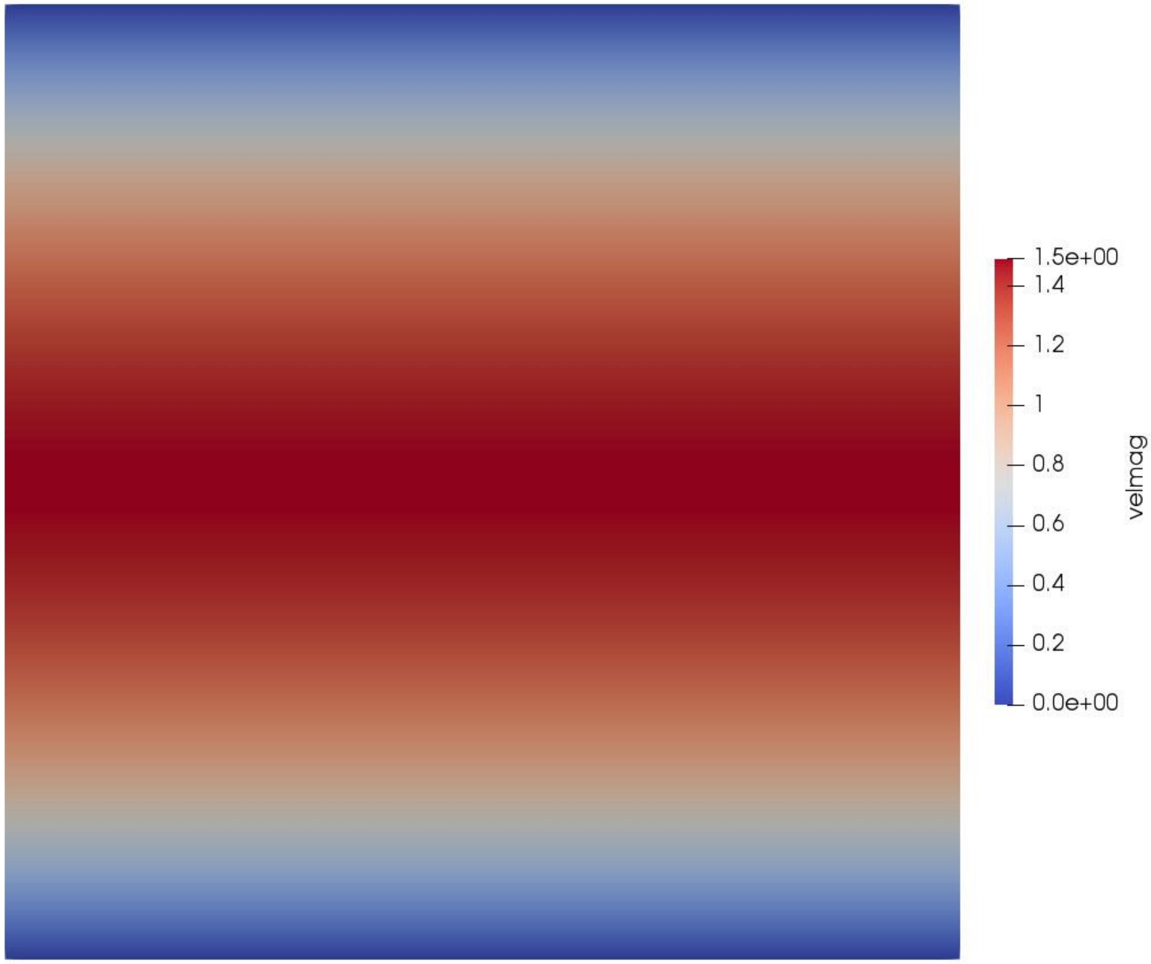
1) Set inlet and outlet velocity profiles from initial guess so that mass is conserved overall.

2) Set a zero streamwise derivative boundary condition at the outlet

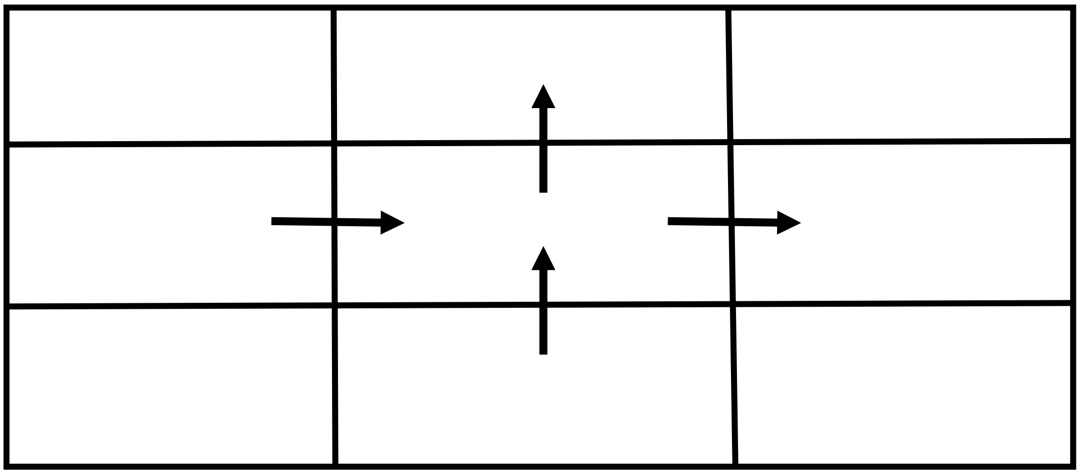
3) Complete an outer iteration, then impose the outlet velocity profile onto the inlet profile.

4) Continue process until flow is fully developed.

See example channel flow next page:


1.5e+00
1.4
1.2
1
0.8
0.6
0.4
0.2
0.0e+00
velmag

Place Solid Object within Flow

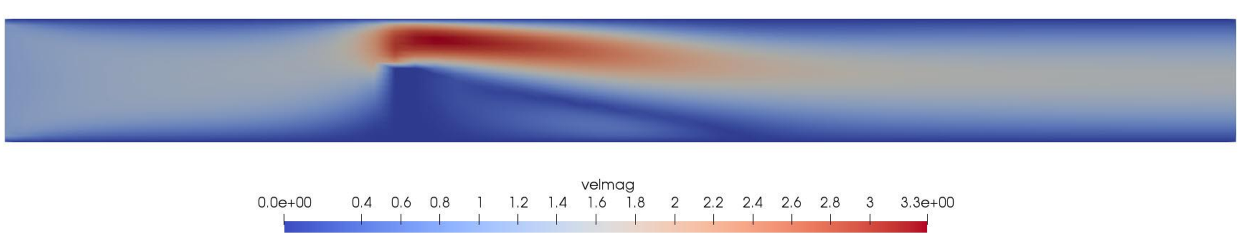


Block out middle cell.

1) In the x-momentum solver set $A_P^u$ to a very large number (10^30) so that $U_P$ does not change from initial guess (which should be zero for a blocked out cell).

2) Same procedure for y-momentum to fix $V_P$ at zero.

Can construct fairly elaborate geometries by this basic approach.




Example of Flow Blockage within Channel Flow (left to right).

Note distance downstream outflow boundary is from blockage.