Section 4: Finite Volume Method for Convection Problems

Governing equation for convection of a scalar:

$$\frac{\partial}{\partial x}(\rho u \phi) + \frac{\partial}{\partial y}(\rho v \phi) = 0$$

or in vector notation

$$\nabla \cdot (\rho \bar{u} \phi) = 0$$

Integrate over a control volume:

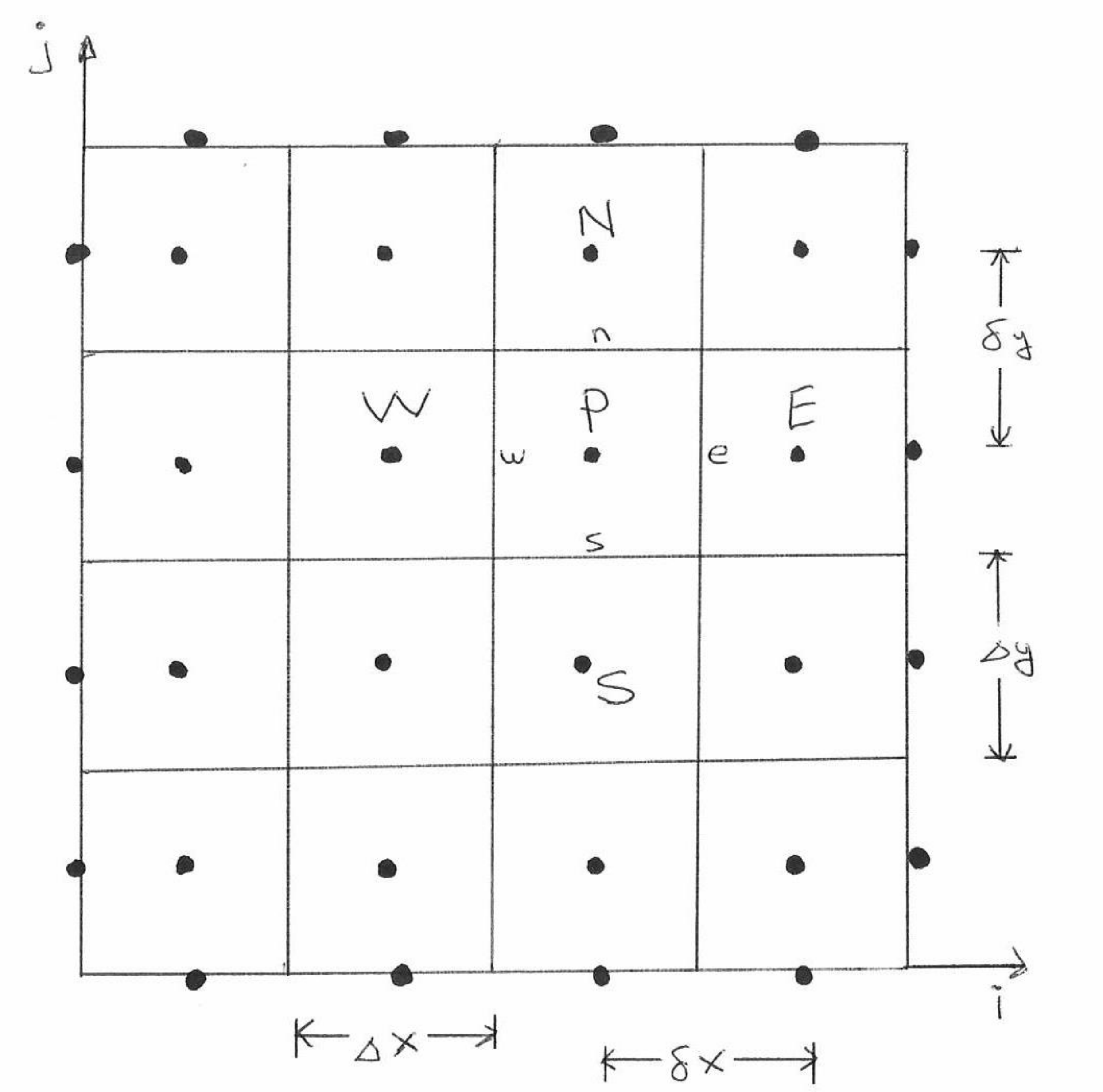
$$\int \nabla \cdot (\rho \bar{u} \phi)\, dV = 0$$

Apply divergence theorem:

$$\int (\rho \bar{u} \phi) \cdot \hat{n}\, dA = 0$$

Evaluate the integral around the control volume:

$$(\rho u \phi|_e - \rho u \phi|_w)\Delta y + (\rho v \phi|_n - \rho v \phi|_s)\Delta x = 0 \qquad \text{(EQ1)}$$



How do we handle $\phi_{e,}\phi_{w,}\phi_{n,}\phi_s$ terms as these are not at cell centers?

Interpolation to cell faces using cell centered values is required.

Perhaps the most intuitive method is to average using neighboring cell center values, generally known as **central differencing** (even though it is a linear interpolation). That is:

$$\phi_e = \frac{1}{2}(\phi_E + \phi_P)$$

$$\phi_w = \frac{1}{2}(\phi_P + \phi_W)$$

$$\phi_n = \frac{1}{2}(\phi_N + \phi_P)$$

$$\phi_s = \frac{1}{2}(\phi_P + \phi_S)$$

If we substitute these expressions into our finite volume equation (EQ1) we get:

$$\dot{m}_e(\phi_E + \phi_P)/2 - \dot{m}_w(\phi_P + \phi_W)/2 + \dot{m}_n(\phi_N + \phi_P)/2 - \dot{m}_s(\phi_S + \phi_P)/2 = 0 \qquad \text{(EQ2)}$$

where we have used the notation $\dot{m}_e = \rho u_e \Delta y$, etc.

Isolating ϕ_P: on the left side:

$$\frac{1}{2}(\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s)\phi_P = -\dot{m}_e\phi_E/2 + \dot{m}_w\phi_W/2 - \dot{m}_n\phi_N/2 + \dot{m}_s\phi_S/2$$

Using notation that is standard in the CFD community, define a set of coefficients as:

$$\tilde{A}_E = -\dot{m}_e/2 \quad \tilde{A}_W = \dot{m}_w/2 \quad \tilde{A}_N = -\dot{m}_n/2 \quad \tilde{A}_S = \dot{m}_s/2$$

so that the final discretized equation may be written in “standard” form as:

$$\tilde{A}_P\phi_P = \tilde{A}_E\phi_E + \tilde{A}_W\phi_W + \tilde{A}_N\phi_N + \tilde{A}_S\phi_S + \tilde{S}_u \qquad \text{(EQ2)}$$

where

$$\tilde{A}_P \equiv \tilde{A}_E + \tilde{A}_W + \tilde{A}_N + \tilde{A}_S + (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s) - \tilde{S}_P$$

To get our equation into this form we subtract and add $\dot{m}_e\phi_P$, and also add and subtract $\dot{m}_w\phi_P$, on the left side. Similarly for the north/south directions.

The $\tilde{S}_u$ and $\tilde{S}_P$ terms are representations for source terms which may be present in the differential equation itself, or arise from the convenient implementation of boundary conditions. For our equation, they are both zero.

Another Interpolation Option:

Alternatively, we can consider an **upwinding** technique. If the flow were from west-to-east, and south-to-north, a simple extrapolation would yield:

$$\phi_e = \phi_P; \qquad \phi_w = \phi_W; \qquad \phi_n = \phi_P; \qquad \phi_s = \phi_S$$

Substituting these extrapolations into (EQ1), we get:

$$\dot{m}_e\phi_P - \dot{m}_w\phi_W + \dot{m}_n\phi_P - \dot{m}_s\phi_S = 0$$

Isolate ϕ_P on the left side:

$$(\dot{m}_e + \dot{m}_n)\phi_P = \dot{m}_w\phi_W + \dot{m}_s\phi_S$$

Then in our standard form:

$$A_E = 0 \quad A_W = \dot{m}_w \quad A_N = 0 \quad A_S = \dot{m}_s$$

However, we need to add and subtract both $\dot{m}_w\phi_P$ and $\dot{m}_s\phi_P$ from the left side in order to put the equation in the standard form:

$$A_P\phi_P = A_E\phi_E + A_W\phi_W + A_N\phi_N + A_S\phi_S + S_u \qquad \text{(EQ3)}$$

where

$$A_P \equiv A_E + A_W + A_N + A_S + (\dot{m}_e - \dot{m}_w + \dot{m}_n - \dot{m}_s) - S_P$$

For our equation, both S_u and S_P are zero, however, these values will be modified along the boundaries as will be shown in an example to follow.

Deferred Correction

Consider the combination of discretized equations:

$EQ3 + \beta(EQ2 - EQ3)$=0

where EQ3 is the complete discretized equation resulting from the lower order (upwinding) interpolation and EQ2 is the discretized equation resulting from the higher order (central) interpolation scheme. The variable β is a "blending" factor. For instance, $\beta = 0$ results in upwinding (a first order method), $\beta = 1$ results in linear interpolation (a second order method), and $0 < \beta < 1$ results in a blending of the two. Making the substitutions:

$$
\begin{aligned}
&(A_P\phi_P - A_E\phi_E - A_W\phi_W - A_N\phi_N - A_S\phi_S - S_u) \\
&\qquad + \beta\big[\big(\tilde{A}_P\phi_P - \tilde{A}_E\phi_E - \tilde{A}_W\phi_W - \tilde{A}_N\phi_N - \tilde{A}_S\phi_S - \tilde{S}_u\big) \\
&\qquad - (A_P\phi_P - A_E\phi_E - A_W\phi_W - A_N\phi_N - A_S\phi_S - S_u)\big]^{OLD} = 0
\end{aligned}
$$

Now we solve for the "red" colored ϕ_p to obtain our iteration equation:

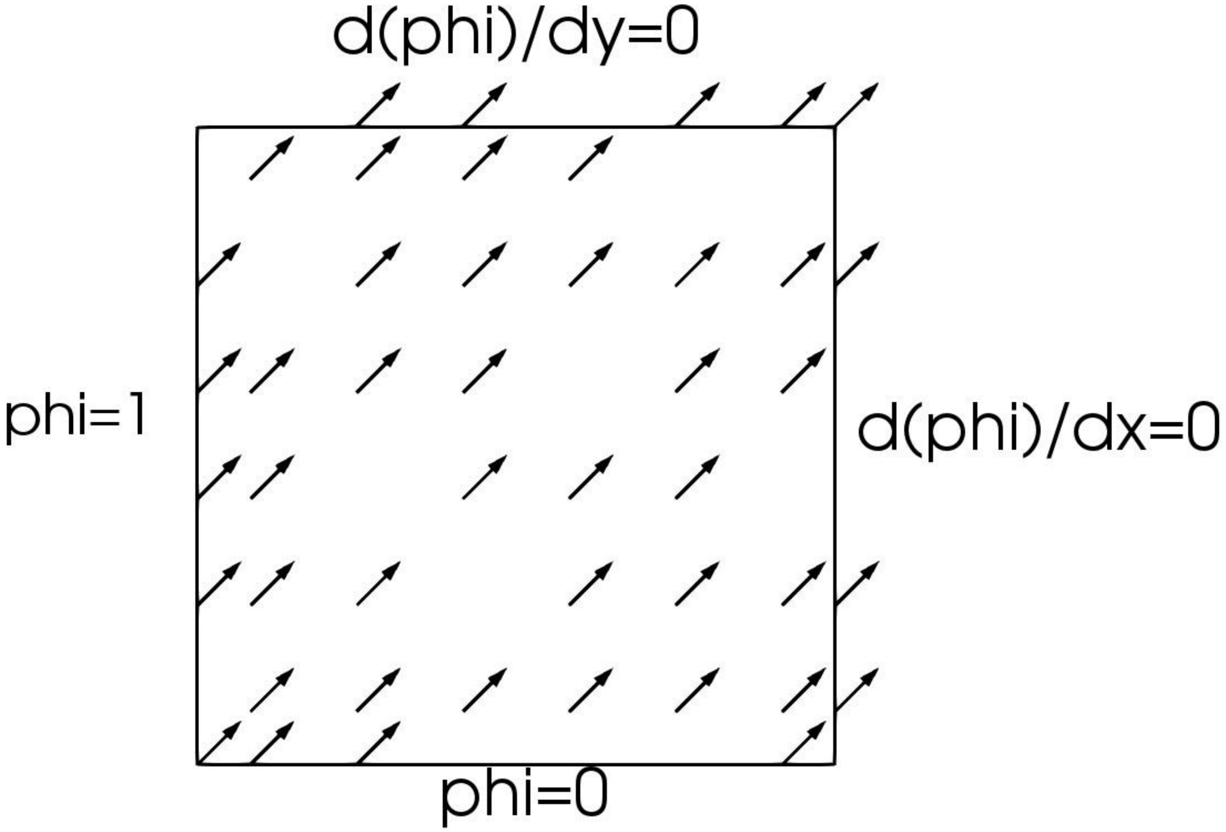
$$\phi_P = (A_E\phi_E + A_W\phi_W + A_N\phi_N + A_S\phi_S + S_u)/A_P - \frac{\beta}{A_P}\big[\big(\tilde{A}_P\phi_P - \tilde{A}_E\phi_E - \tilde{A}_W\phi_W - \tilde{A}_N\phi_N - \tilde{A}_S\phi_S - \tilde{S}_u\big) - (A_P\phi_P - A_E\phi_E - A_W\phi_W - A_N\phi_N - A_S\phi_S - S_u)\big]^{OLD} \qquad \text{(EQ4)}$$

EXAMPLE PROBLEM

Consider the convection of a step profile in a uniform incompressible flow oblique to the grid lines the domain over the region 0<=*x*<=1 and 0<=*y*<=1 as shown in the figure below. Let *u*=*v*=1 everywhere. The boundary conditions are $\phi = 1$ over the the west boundary, $\phi = 0$ on the south boundary, $\partial\phi/\partial x = 0$ on the east boundary and $\partial\phi/\partial y = 0$ on the north boundary. If we neglect the effects of diffusion the governing equation becomes:

$$\frac{\partial}{\partial x}(\varrho u\phi) + \frac{\partial}{\partial y}(\rho v\phi) = 0$$

We want to implement the deferred correction method to solve the problem and obtain solutions for values of the blending factor given by 0.0 (representing pure 1st order upwind), 0.9 (representing a blending between 1st order upwind and 2nd order central), and 1.0 (representing pure 2nd order central).


d(phi)/dy=0
phi=1
d(phi)/dx=0
phi=0

The coefficients in EQ4 will be modified to account for the boundary conditions. In particular, for the **first order upwinding** scheme, with flow from west-to-east:

West Boundary (fixed, Dirichlet boundary condition)

(we will ignore north/south terms as they remain as previously defined)

$$\dot{m}_e \phi_e - \dot{m}_w \phi_w = 0$$

Apply upwinding:

$$\dot{m}_e \phi_P - \dot{m}_w \phi_{wbc} = 0$$

$$\dot{m}_e \phi_P = \dot{m}_w \phi_{wbc}$$

Hence, $A_E = A_W = 0$.

$$S_p = -\dot{m}_e$$

$$S_u = \dot{m}_w \phi_{wbc}$$

$$A_P \equiv A_E + A_W - S_P$$

East Boundary (derivative, Neumann boundary condition)

$$\dot{m}_e \phi_e - \dot{m}_w \phi_w = 0$$

For zero derivative, $\phi_e = \phi_P$, so that

$$\dot{m}_e \phi_P = \dot{m}_w \phi_W$$

In our standard form:

$$\dot{m}_w \phi_P + (\dot{m}_e - \dot{m}_w)\phi_P = \dot{m}_w \phi_W$$

So that:

$$S_u = 0; \quad S_p = -(\dot{m}_e - \dot{m}_w); \qquad A_W = \dot{m}_w; \qquad A_E = 0$$

South Boundary

$S_u = \dot{m}_s \phi_{sbc}; \quad S_p = -\dot{m}_n; \qquad A_N = A_S = 0$

North Boundary

$S_u = 0; \quad S_p = -(\dot{m}_n - \dot{m}_s); \qquad A_S = \dot{m}_s; \qquad A_N = 0$

For the 2nd order central scheme (independent of flow direction):

West Boundary

$$\dot{m}_e\phi_e - \dot{m}_w\phi_w = 0$$

$$\frac{\dot{m}_e}{2}(\phi_P + \phi_E) = \dot{m}_w\phi_{wbc}$$

$$\frac{\dot{m}_e}{2}\phi_P = -\frac{\dot{m}_e}{2}\phi_E + \dot{m}_w\phi_{wbc}$$

$$-\frac{\dot{m}_e}{2}\phi_P + \dot{m}_e\phi_P = -\frac{\dot{m}_e}{2}\phi_E + \dot{m}_w\phi_{wbc}$$

So that:

$S_u = \dot{m}_w\phi_{wbc};$ $S_p = -\dot{m}_e$

$A_W = 0;$ $A_E = -\frac{\dot{m}_e}{2}$

East Boundary

$$\dot{m}_e\phi_e - \dot{m}_w\phi_w = 0$$

$$\dot{m}_e\phi_P - \frac{\dot{m}_w}{2}(\phi_P + \phi_W) = 0$$

$$\left(\dot{m}_e - \frac{\dot{m}_w}{2}\right)\phi_P = \frac{\dot{m}_w}{2}\phi_W$$

$$\frac{\dot{m}_w}{2}\phi_P + (\dot{m}_e - \dot{m}_w)\phi_P = \frac{\dot{m}_w}{2}\phi_W$$

So that:

$S_u = 0;$ $S_p = -(\dot{m}_e - \dot{m}_w)$

$A_W = \frac{\dot{m}_w}{2};$ $A_E = 0$

South Boundary

$S_u = \dot{m}_s \phi_{sbc}; \qquad S_p = -\dot{m}_n$

$A_S = 0; \qquad A_N = -\frac{\dot{m}_n}{2};$

North Boundary

$S_u = 0; \quad S_p = -(\dot{m}_n - \dot{m}_s)$

$A_S = \frac{\dot{m}_s}{2}; \qquad A_N = 0$

Corner Cells (Volumes)

Let’s look at the northwest cell.

This can be considered a combination of west boundary results and north boundary results.

We then find for the upwinding coefficients:

$$S_u = \dot{m}_w \phi_{wbc} + 0$$

$$S_P = -\dot{m}_e - (\dot{m}_n - \dot{m}_s)$$

$$A_W = A_N = 0$$

And for the central difference coefficients, the same results:

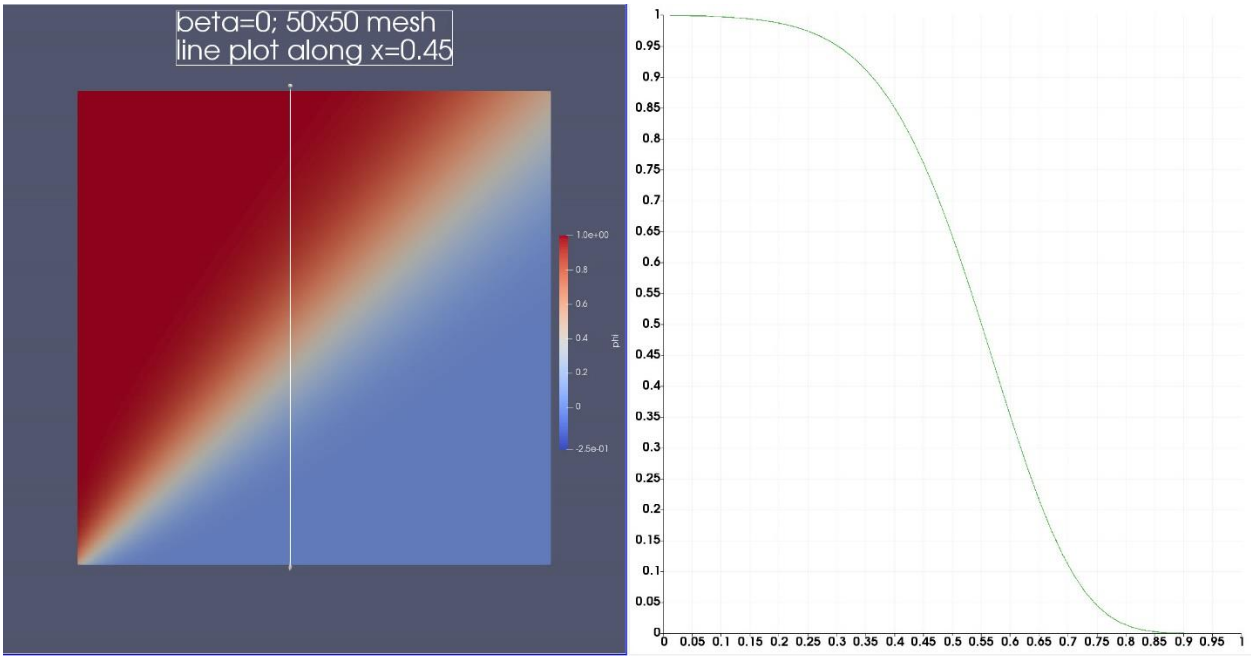
$$S_u = \dot{m}_w \phi_{wbc} + 0$$

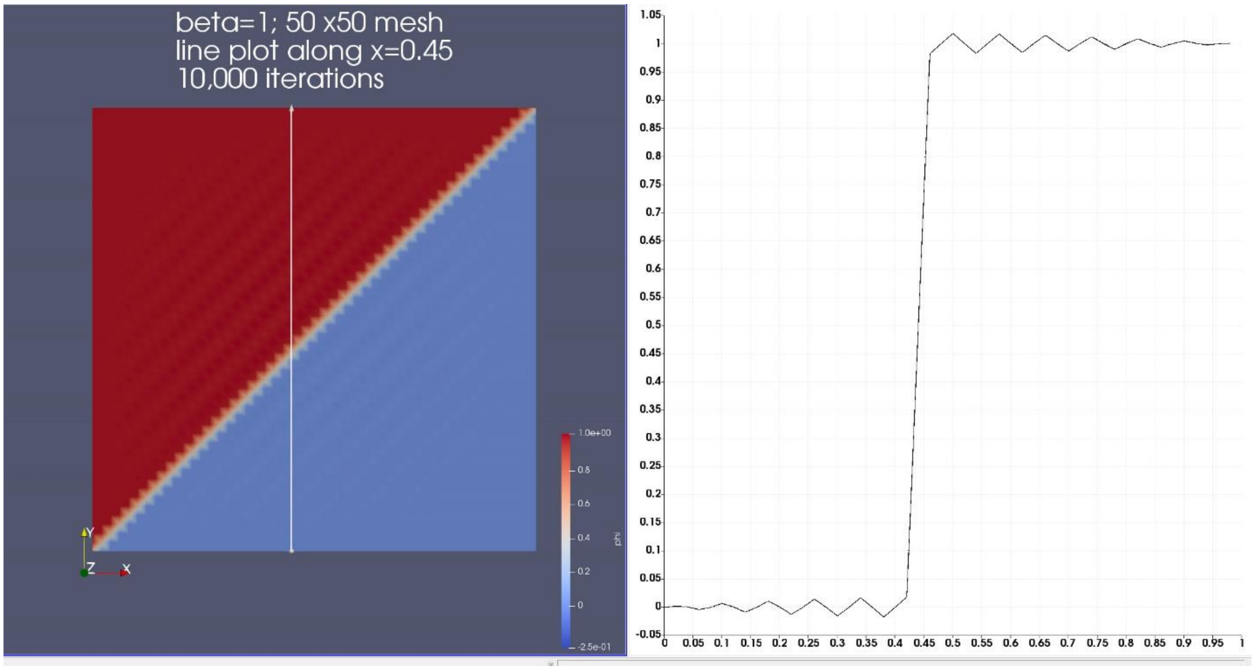
$$S_P = -\dot{m}_e - (\dot{m}_n - \dot{m}_s)$$

$$A_W = A_N = 0$$

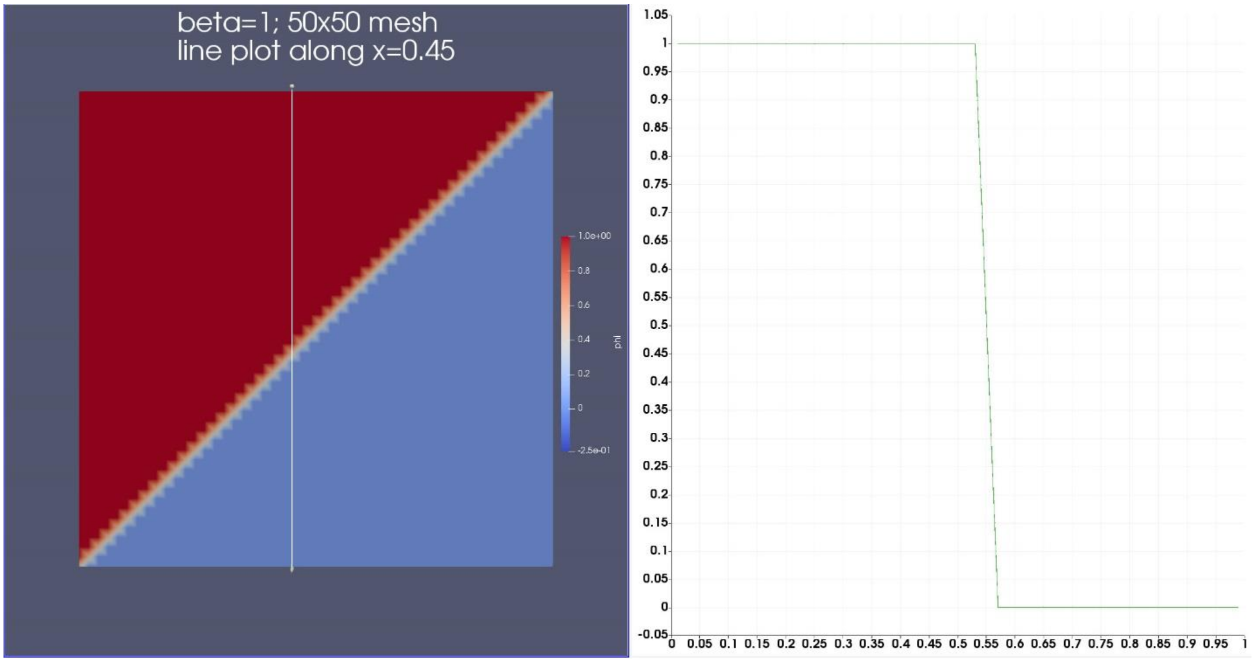
You can find the other corner cells by inspecting the code which is included to solve this example problem.

RESULTS (with phi=1 along entire west boundary)

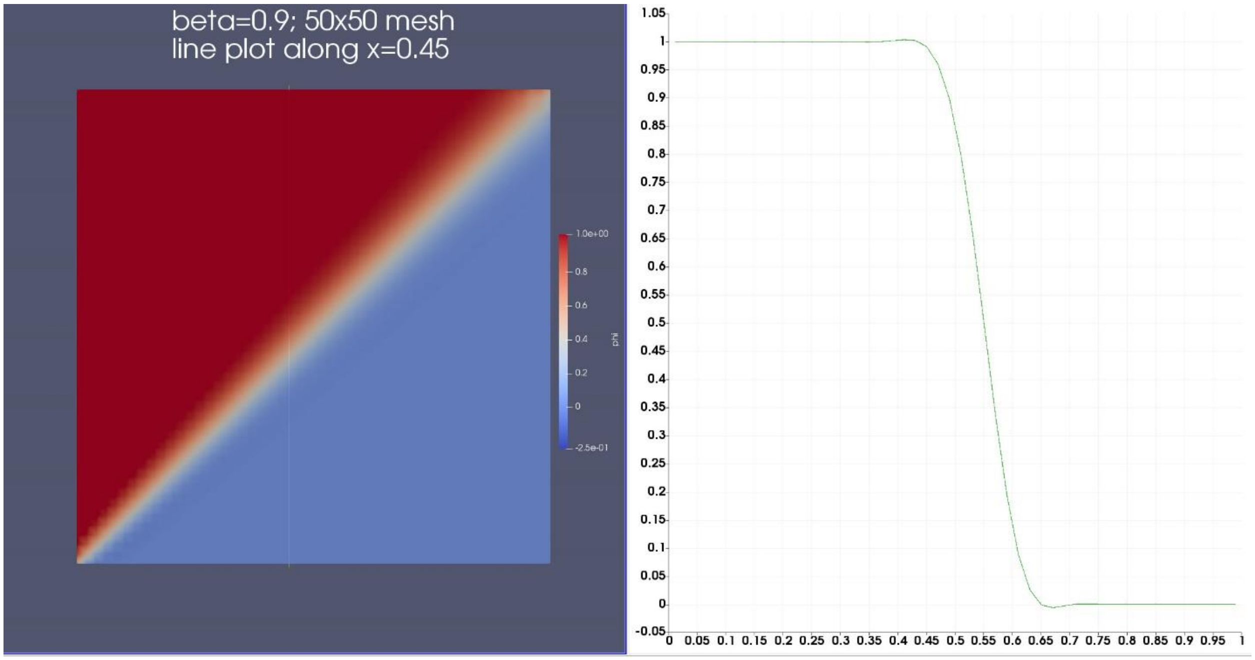




2nd order central at 10,000 iterations

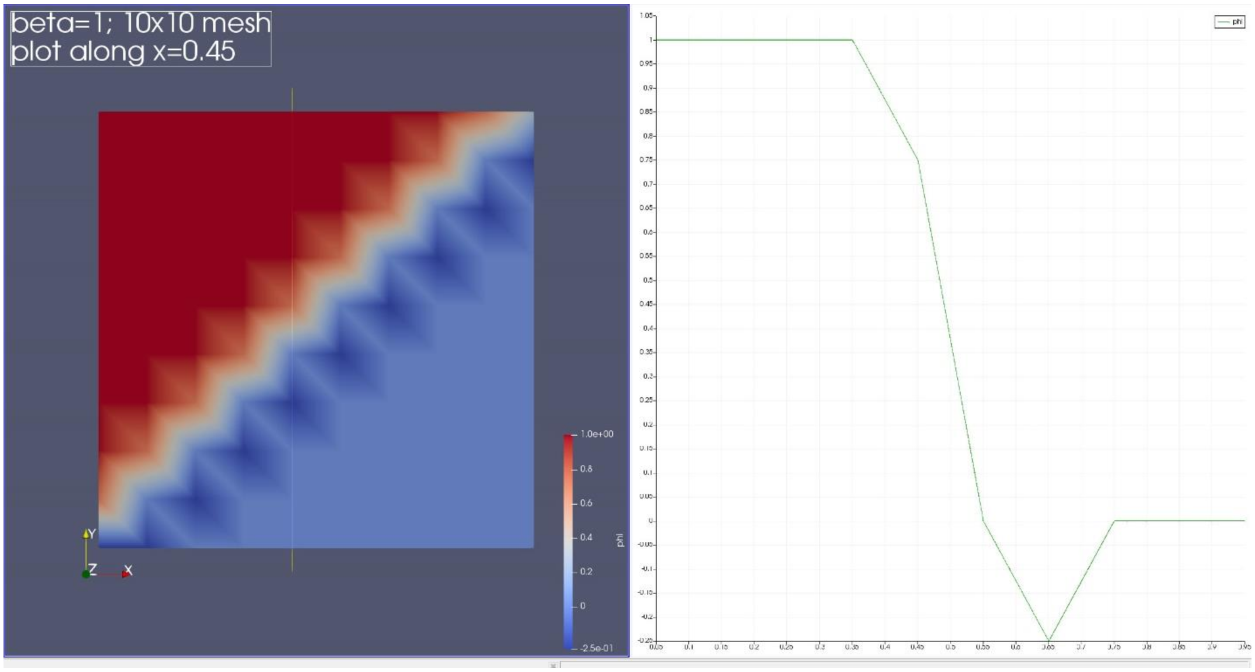


(2nd order central at 100,000 iterations)

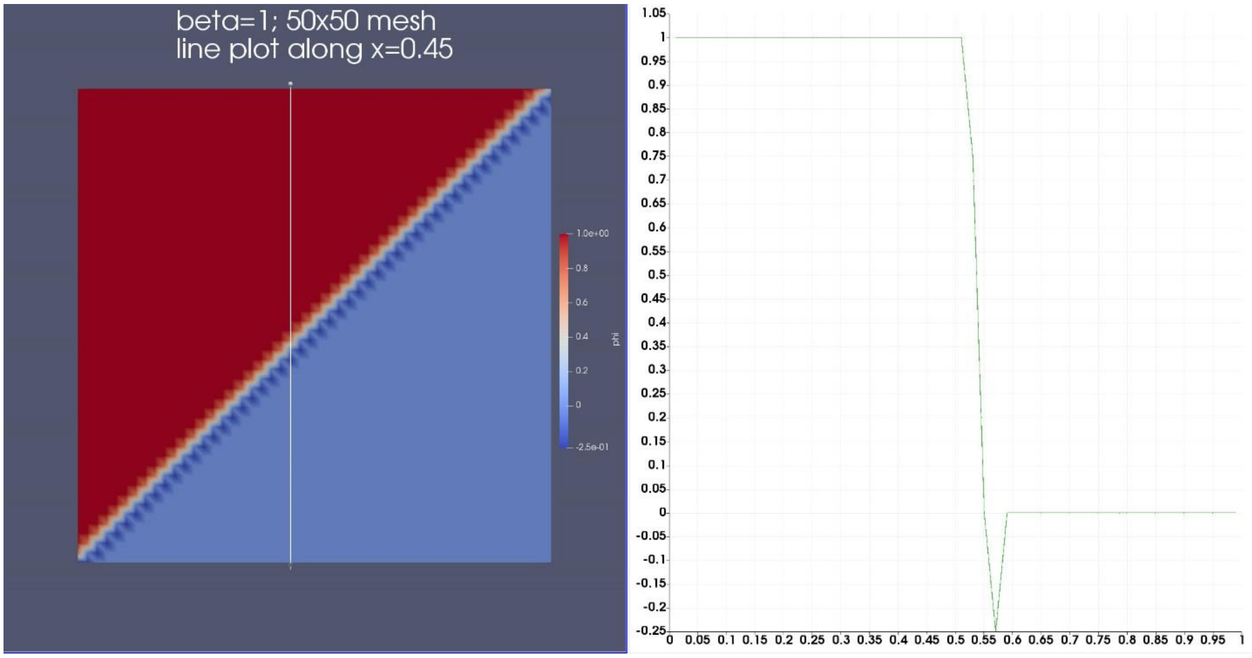


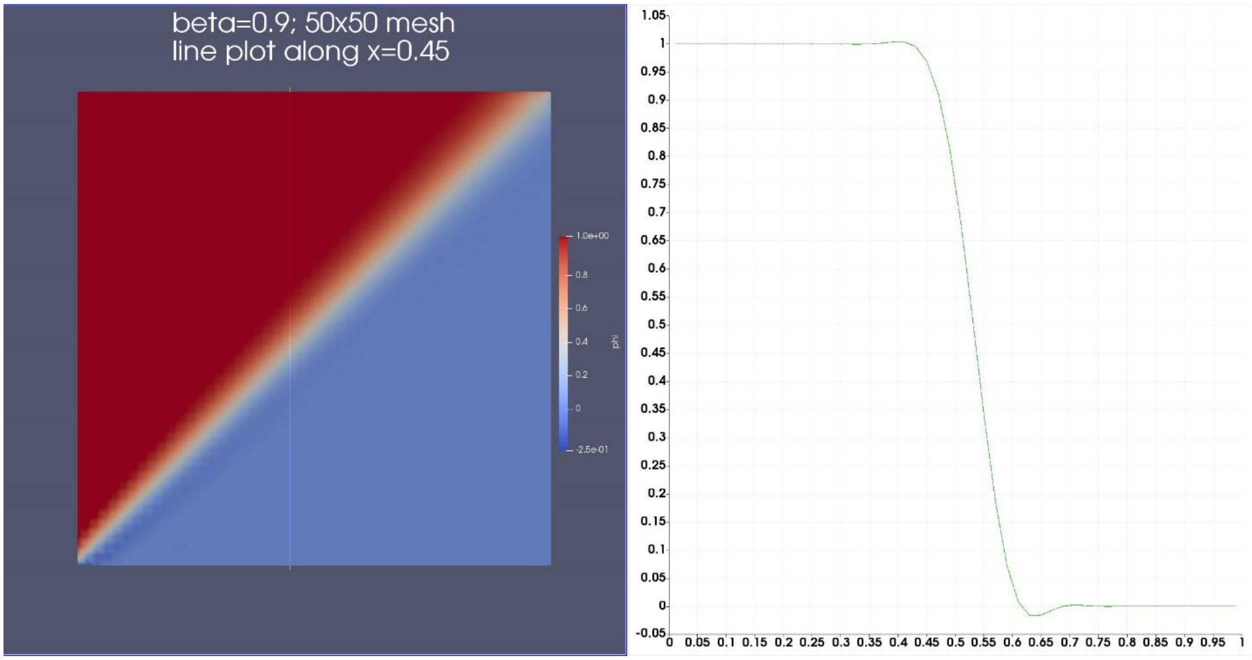
(converged blended result)

ADJUST WEST BOUNDARY CONDITIONS SO THAT **PHI=0 ON WEST SIDE OF SOUTHWEST CORNER CELL**



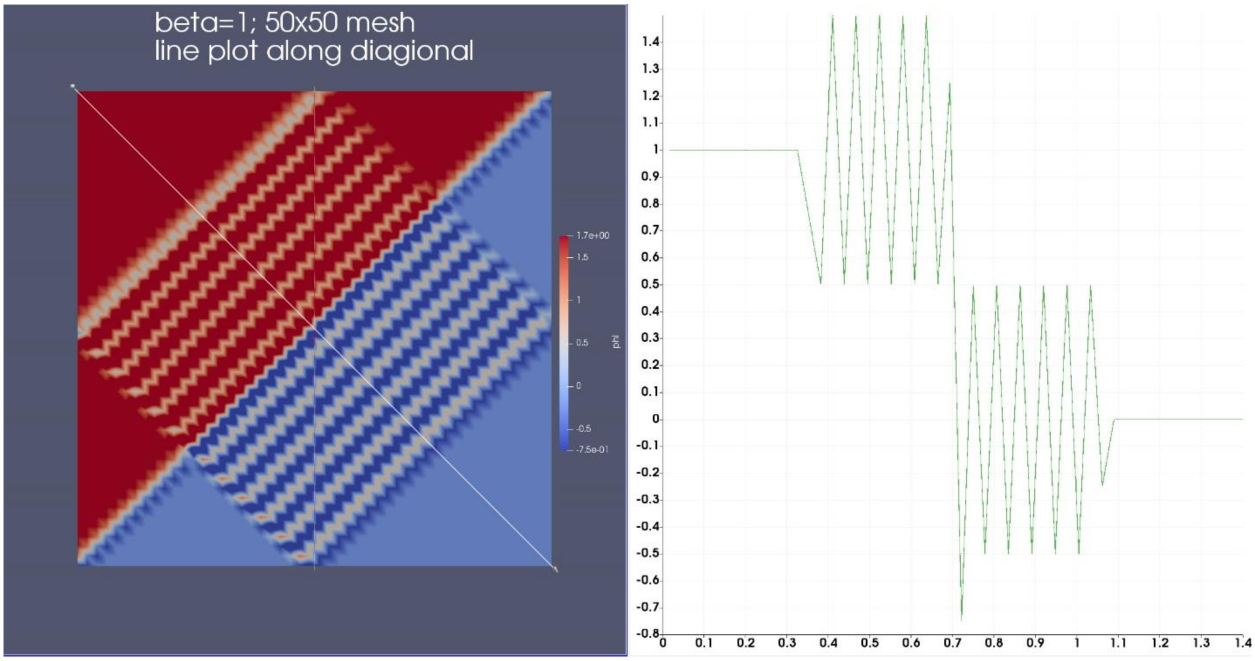
(same result as in Ferziger and Peric text: fig. 4.8, edition 3)

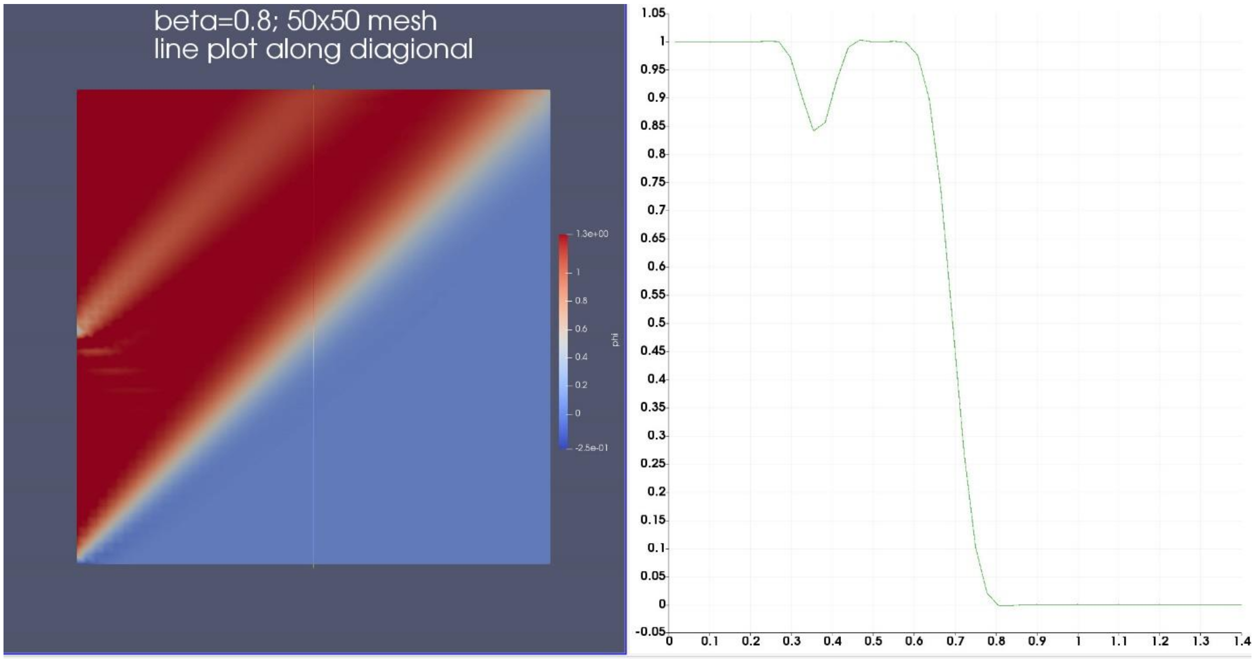




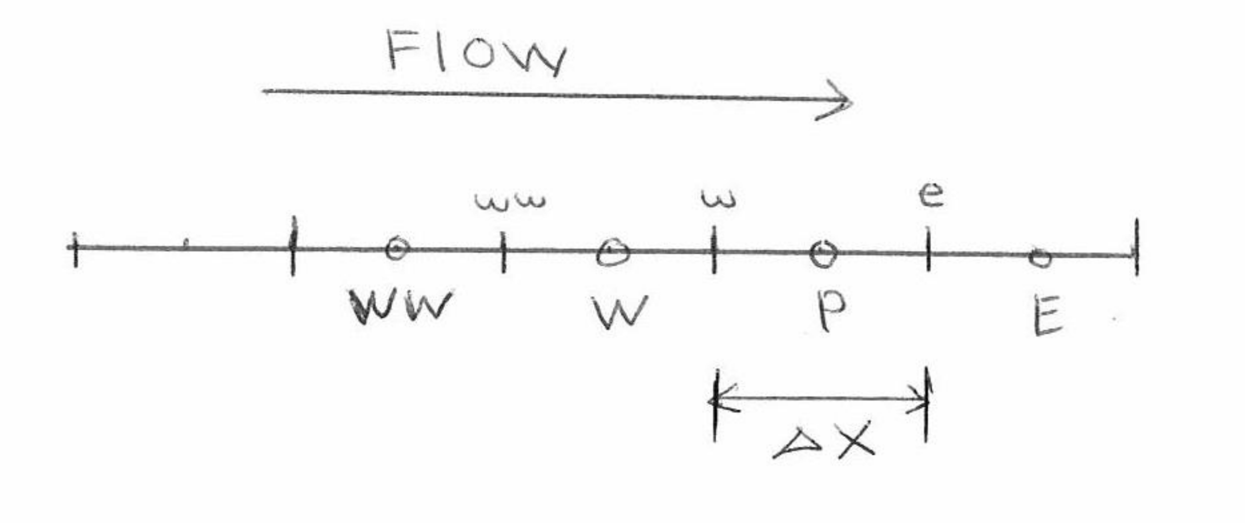
(blended solution)

MOVE PHI=0 BC CELL TO MIDPOINT OF WEST BOUNDARY





OTHER INTERPOLATION SCHEMES



These are all “upwinding” methods; we will discuss with flow in the west-to-east direction. Similar procedures provide interpolations for flows east-to-west.

Linear Upwinding

Express ϕ_e in terms of ϕ_P and ϕ_W for flow direction west-to-east.

Let the origin (x=0) of our diagram above sit at the east face.

Then express ϕ in the form:

$$\phi = a\,x + b$$

So that:

$$\phi_P = -a\,(\Delta x/2) + b$$

$$\phi_W = -a\,(3\Delta x/2) + b$$

Solving for the unknowns *a* and *b*, find:

$$a = \frac{\phi_P - \phi_W}{\Delta x}$$

$$b = \frac{1}{2}(3\phi_P - \phi_W)$$

But, since the east face sits at *x*=0, $\phi_e = b$, or:

$$\phi_e = \frac{3}{2}\phi_P - \frac{1}{2}\phi_W$$

Similarly,

$$\phi_w = \frac{3}{2}\phi_W - \frac{1}{2}\phi_{WW}$$

Applying this interpolation to our model equation:

$$\dot{m}_e\phi_e - \dot{m}_w\phi_w = 0$$

$$\dot{m}_e\left(\frac{3}{2}\phi_P - \frac{1}{2}\phi_W\right) - \dot{m}_w\left(\frac{3}{2}\phi_W - \frac{1}{2}\phi_{WW}\right) = 0$$

$$\frac{3}{2}\dot{m}_e\phi_P = \frac{1}{2}\dot{m}_e\phi_W + \frac{3}{2}\dot{m}_w\phi_W - \frac{1}{2}\dot{m}_w\phi_{WW}$$

1) add & subtract $\frac{3}{2}\dot{m}_w\phi_P$ from left side

2) subtract and add $\dot{m}_e\phi_P$ from left side

3) subtract and add $\frac{1}{2}\dot{m}_w\phi_P$ from left side

This gives:

$$\left(\frac{3}{2}\dot{m}_w - \dot{m}_e - \frac{1}{2}\dot{m}_w + \frac{3}{2}\dot{m}_e\right)\phi_P + \left(-\frac{3}{2}\dot{m}_w + \dot{m}_e + \frac{1}{2}\dot{m}_w\right)\phi_P = \frac{1}{2}\dot{m}_e\phi_W + \frac{3}{2}\dot{m}_w\phi_W - \frac{1}{2}\dot{m}_w\phi_{WW}$$

Simplify:

$$\left(\dot{m}_w + \frac{1}{2}\dot{m}_e\right)\phi_P + (\dot{m}_e - \dot{m}_w)\phi_P$$
$$= \frac{1}{2}\dot{m}_e\phi_W + \frac{3}{2}\dot{m}_w\phi_W - \frac{1}{2}\dot{m}_w\phi_{WW}$$

where:

$A_E = 0$

$A_W = \frac{1}{2}\dot{m}_e + \frac{3}{2}\dot{m}_w$

$A_{WW} = -\frac{1}{2}\dot{m}_w$

$S_P = 0$

$S_u = 0$

$A_P \equiv A_W + A_{WW} + (\dot{m}_e - \dot{m}_w) - S_P$

$A_P\phi_P = A_W\phi_W + A_{WW}\phi_{WW}$

Quadratic Interpolation for Convective Kinematics (QUICK) scheme

Using a procedure similar to that above (i.e., let $\phi = ax^2 + b\ x + c$ and write at points E, P, and W for the east face) we find, for flow from west-to-east:

$\phi_e = \frac{1}{8}(3\phi_E + 6\phi_P - \phi_W)$

$\phi_w = \frac{1}{8}(3\phi_P + 6\phi_W - \phi_{WW})$

TRUNCATION ERRORS

1) upwinding

Taylor series expansion about ϕ_P.

$$\phi_e = \phi_P + \left.\frac{d\phi}{dx}\right|_P \frac{\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_P \frac{\Delta x^2}{2!} + O(\Delta x^3)$$

In our approximation, $\phi_e = \phi_P$, so the truncation error is of $O(\Delta \mathrm{x})$.

2) **linear interpolation (or central "differencing")**

$$\phi_E = \phi_e + \left.\frac{d\phi}{dx}\right|_e \frac{\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{(\Delta x/2)^2}{2!} + \left.\frac{d^3\phi}{dx^3}\right|_e \frac{(\Delta x/2)^3}{3!} + O(\Delta x^4)$$

$$\phi_P = \phi_e - \left.\frac{d\phi}{dx}\right|_e \frac{\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{(\Delta x/2)^2}{2!} - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{(\Delta x/2)^3}{3!} + O(\Delta x^4)$$

Add these two equations:

$$\phi_E + \phi_P = 2\phi_e + 2\left.\frac{d^2\phi}{dx^2}\right|_e \frac{(\Delta x/2)^2}{2!} + O(\Delta x^4)$$

Or

$$\phi_e = \frac{\phi_E + \phi_P}{2} + O(\Delta x^2)$$

so the truncation error is $O(\Delta x^2)$.

3) **linear upwind interpolation**

$$\phi_P = \phi_e - \left.\frac{d\phi}{dx}\right|_e \frac{\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{(\Delta x/2)^2}{2!} - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{(\Delta x/2)^3}{3!} + O(\Delta x^4)$$

$$\phi_W = \phi_e - \left.\frac{d\phi}{dx}\right|_e \frac{3\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{(3\,\Delta x/2)^2}{2!} - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{(3\,\Delta x/2)^3}{3!} + O(\Delta x^4)$$

Multiply the first equation by 3/2 and the second by 1/2. Then subtract the second from the first:

$$\frac{3}{2}\phi_P - \frac{1}{2}\phi_W = \phi_e + O(\Delta x)^2$$

Consequently, the truncation error is of $O(\Delta x)^2$.

4) QUICK interpolation

Similar procedure, find truncation error of $O(\Delta x)^3$.

Generalization of upwind interpolation schemes

1st order upwinding

$$\phi_e = \phi_P$$

2nd order linear (central)

$$\phi_e = \frac{1}{2}(\phi_E + \phi_P)$$

2nd order linear upwinding

$$\phi_e = \frac{3}{2}\phi_P - \frac{1}{2}\phi_W$$

3rd order QUICK

$$\phi_e = \frac{1}{8}(3\phi_E + 6\phi_P - \phi_W)$$

These can all be written in the form

$$\phi_e = \phi_P + correction$$

That is:

Linear (central differencing)

$$\phi_e = \phi_P + \frac{1}{2}(\phi_E - \phi_P)$$

Linear upwinding:

$$\phi_e = \phi_P + \frac{1}{2}(\phi_P - \phi_W)$$

QUICK

$$\phi_e = \phi_P + \frac{1}{8}(3\phi_E - 2\phi_P - \phi_W)$$

Consider the generalization:

$$\phi_e = \phi_P + \frac{1}{2}\Psi(\phi_E - \phi_P)$$

Upwinding in this form:

$$\Psi = 0$$

Linear interpolation (central differencing) in this form:

$$\Psi = 1$$

Linear upwind interpolation in this form:

$$\phi_e = \phi_P + \tfrac{1}{2}(\phi_P - \phi_W)$$

or

$$\phi_e = \phi_P + \frac{1}{2}\frac{\phi_P - \phi_W}{\phi_E - \phi_P}(\phi_E - \phi_P)$$

So that

$$\Psi = \frac{\phi_P - \phi_W}{\phi_E - \phi_P}$$

QUICK interpolation in this form:

$$\Psi = \frac{1}{4}\left[3 + \frac{\phi_P - \phi_W}{\phi_E - \phi_P}\right]$$

It appears that Ψ is a function of

$$r \equiv \frac{\phi_P - \phi_W}{\phi_E - \phi_P}$$

Which is a representation of the downstream solution gradient over the upstream solution gradient.

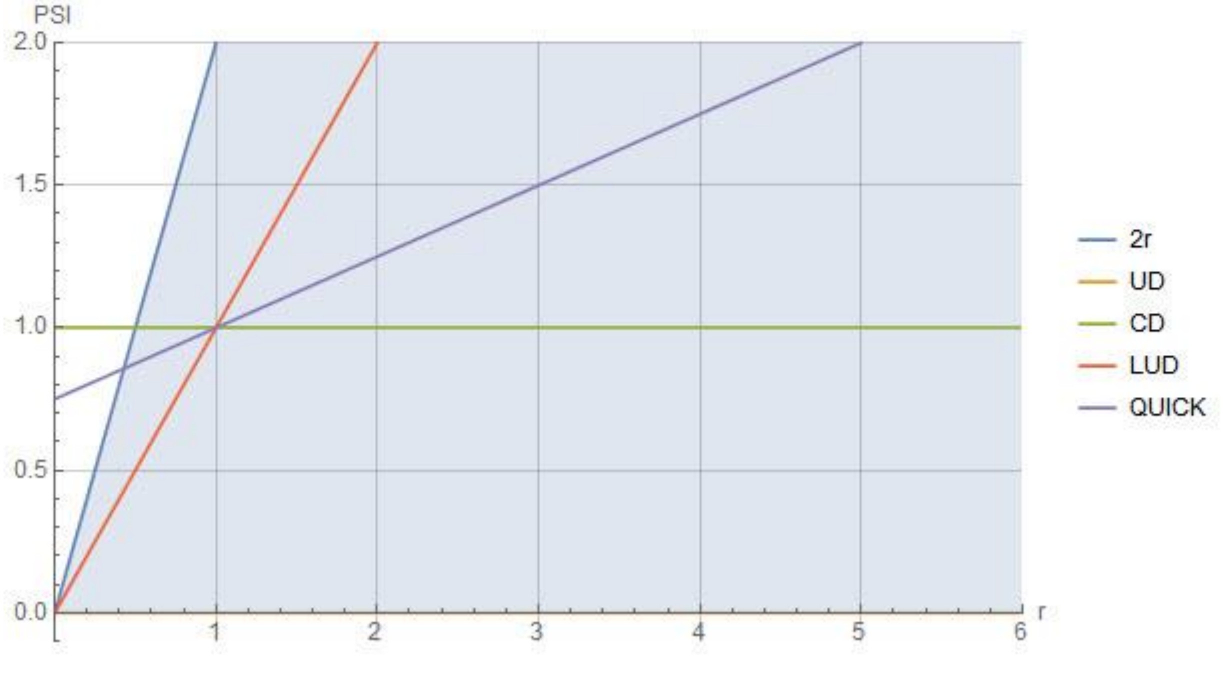
Hence, in general we can write:

$$\phi_e = \phi_P + \frac{1}{2}\Psi(r)(\phi_E - \phi_P)$$

Where for the different interpolations:

Upwinding	$\Psi(r) = 0$
Linear interpolation	$\Psi(r) = 1$
Linear upwind interpolation	$\Psi(\mathrm{r}) = r$
QUICK	$\Psi(\mathrm{r}) = (3 + r)/4$

The shaded area below represents a region where the schemes satisfy a **total variation diminishing** (TVD) requirement. In essence, in this region the schemes will not produce local extrema, or accentuate existing local extrema.

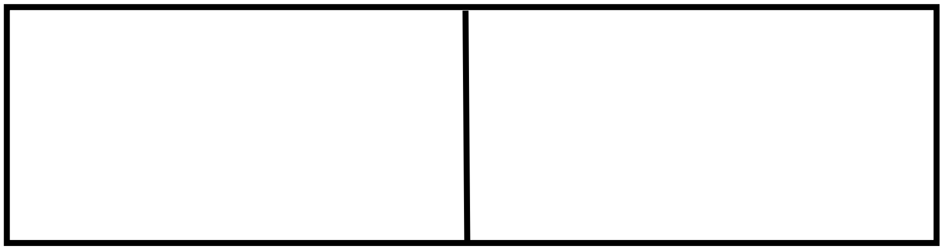


Upwind differencing always remains in shaded area.
Central differencing leaves the shaded area for $r < 0.5$
Linear upwind interpolation leaves the shaded area for $r > 2$.
QUICK leaves the shaded area for $r < 3/7$ or $r > 5$

Requirements for an Interpolation Scheme

Interpolation schemes are not arbitrary. In fact, it is highly desirable that the interpolation scheme satisfy several core properties.

Conserve fluxes at cell interfaces (conservativeness)?



Consider the two adjacent cells above and the calculation of $(\rho U \phi)$ at the interface. Question: is it consistent?

Next, consider the same question for the diffusion term $\left(\Gamma \frac{d\phi}{dx}\right)$.

Promote convergence during iterative process (boundedness)?

a) Scarborough criterion (or diagional dominance).

Recall, in general for a one-dimensional problem:

$A_P \equiv A_E + A_W + (\dot{m}_e - \dot{m}_w) - S_P$

b) Like sign on coefficients (A_E, A_W, A_N, A_S)?

Take flow direction into account (transportiveness)?

Stability Issues for Combined Convection/Diffusion Problems

Further discussion on the like-sign coefficient requirement.

Consider a 1D convection/diffusion problem such as:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)$$

If we apply the finite volume discretization to this equation using 1st order upwinding on the convection terms (flow west to east) we get:

$$\dot{m}_e \phi_P - \dot{m}_w \phi_W = \Delta y \left(\Gamma_e \frac{\phi_E - \phi_P}{(\delta x)_e} - \Gamma_w \frac{\phi_P - \phi_W}{(\delta x)_w}\right)$$

For this example I would like to write $\dot{m}_e$ as $(\rho u)_e \Delta y$ and similarly for the west face term. The above equation then becomes:

$$(\rho u)_e \phi_P - (\rho u)_w \phi_W = \left(\Gamma_e \frac{\phi_E - \phi_P}{(\delta x)_e} - \Gamma_w \frac{\phi_P - \phi_W}{(\delta x)_w}\right)$$

Arranging into our standard form:

$$\left(\rho u_w + \frac{\Gamma_e}{\delta x_e} + \frac{\Gamma_w}{\delta x_w}\right)\phi_P + (\rho u_e - \rho u_w)\phi_P$$
$$= \rho u_w \phi_W + \frac{\Gamma_e}{\delta x_e}\phi_E + \frac{\Gamma_w}{\delta x_w}\phi_W$$

Where $A_E = \frac{\Gamma_e}{\delta x_e}$ and $A_W = \rho u_w + \frac{\Gamma_w}{\delta x_w}$

In this case both coefficients are always positive for flow west to east.

What about linear interpolation for the convection terms?

We end up with the discretized equation:

$$\left(-\frac{\rho u_e}{2} + \frac{\rho u_w}{2}\phi_W + \frac{\Gamma_e}{\delta x_e} + \frac{\Gamma_w}{\delta x_w}\right)\phi_P + (\rho u_e - \rho u_w)\phi_P$$
$$= -\frac{\rho u_e}{2}\phi_E + \frac{\rho u_w}{2}\phi_W + \frac{\Gamma_e}{\delta x_e}\phi_E + \frac{\Gamma_w}{\delta x_w}\phi_W$$

In this case

$$A_E = -\frac{\rho u_e}{2} + \frac{\Gamma_e}{\delta x_e}$$

$$A_W = \frac{\rho u_w}{2} + \frac{\Gamma_w}{\delta x_w}$$

What is the issue? Look carefully at A_E.

Turns out that for A_E to remain positive (as A_W is always positive)

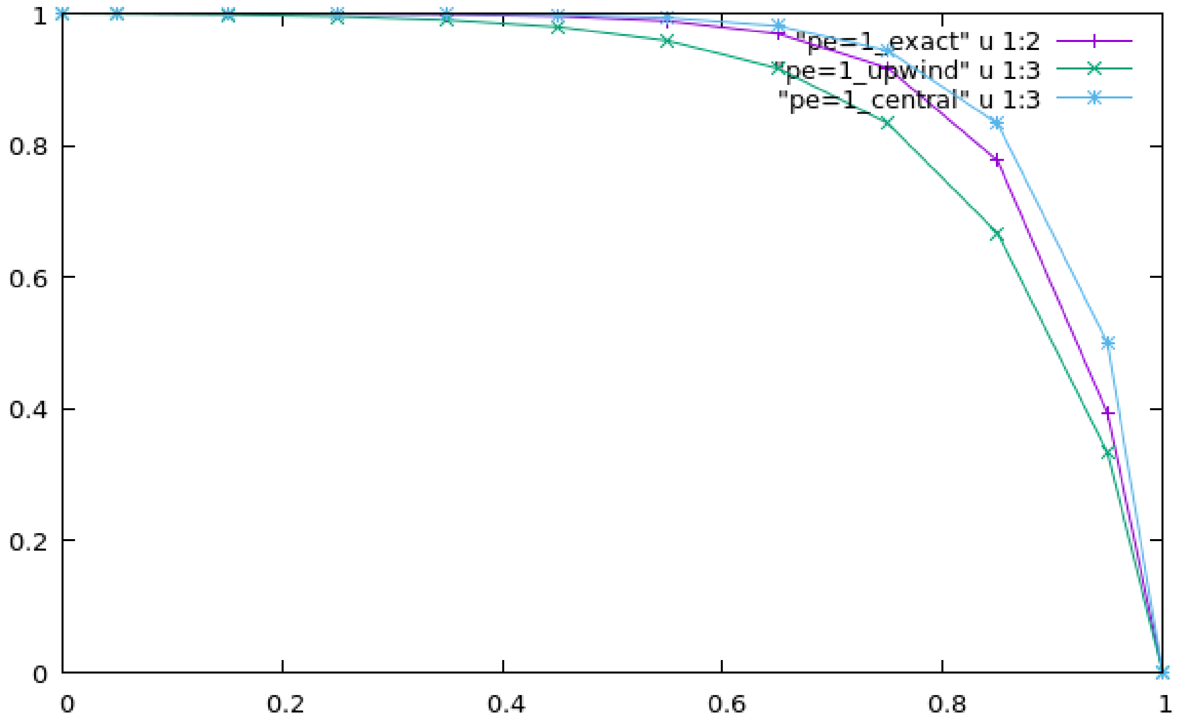
$$\frac{\rho u_e \delta x_e}{\Gamma_e} < 2$$

This is a cell Reynolds number (or Peclet number) restriction for convection/diffusion problems. The exact value is a function of the interpolation scheme.

First-order upwinding does not suffer from this, but suffers from low accuracy.

Test Problem: phi=1 on west inlet boundary; phi=0 on east outlet boundary. Look at results for Pe=1 and Pe=5.

Peclet Number = 1



Peclet Number = 5