How to implement convective heat transfer boundary condition in the diffusion equation (Section 3 in the notes.)

Let's look at a west boundary to be consistent with the notes. We will let ϕ represent temperature.

The convective boundary condition is given as (i.e., conduction balances convection at the wall):

$$-\Gamma \frac{d\phi}{dx}\bigg|_w = -h(\phi_{wb} - \phi_{\infty})$$

(We denote the temperature on the *west* boundary as ϕ_{wb}, which is an unknown and will require an initial guess for the iterative solution. The thermal conductivity is Γ and the convective heat transfer coefficient is *h*.)

Discretize the derivative term:

$$-\Gamma \frac{\phi_P - \phi_{wb}}{dx/2} = -h(\phi_{wb} - \phi_{\infty})$$

Rearrange (solve for ϕ_{wb}):

$$\phi_{wb} = \frac{\frac{2\Gamma\phi_P}{dx} + h\phi_\infty}{\frac{2\Gamma}{dx} + h}$$

Look at the limiting behaviors. For instance, as h gets very large, $\phi_{wb} \to \phi_\infty$. As h goes to zero, $\phi_{wb} \to \phi_p$. Similarly, for limits on gamma.

To implement this, simply substitute the equation for ϕ_{wb} for ϕ_{bc} in the notes (i.e., Section 3; Boundaries; A) Fixed boundary conditions). In other words, we implement the solution procedure by fixing the wall temperature.

However, since the wall temperature is an unknown, you will need to update the values for S_u after every set of iterations. This is because as the value of ϕ_P in the wall adjacent cell changes, ϕ_{wb} will change (see our equation above). After sufficient iterations (upon iterative convergence) the changes will go to zero.

This approach “lags” the boundary temperature, but should be very straightforward to implement in the code.