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Advanced Kalman Filtering and Sensor Fusion

Unscented Kalman Filter Summary

Problem:

$$\hat{x}_{i|j} = \arg\min_{\hat{x}_{i|j} \in \mathcal{R}^n} E\left[\left(x_i - \hat{x}_{i|j}\right) \left(x_i - \hat{x}_{i|j}\right)^T \middle| z_1, \cdots, z_j \right]$$

System Model:

$$x_k = \mathbf{f}(x_{k-1}, u_k, w_k)$$
$$z_k = \mathbf{h}(x_k, v_k)$$

Assumptions:

$$\underbrace{\begin{aligned} w_k &\sim N(0, \mathbf{Q}_k) \\ v_k &\sim N(0, \mathbf{R}_k) \end{aligned}}_{\text{Gaussian Distribution with zero mean and given covariance matrix}}$$

$$\left.\begin{aligned} E(w_k w_j^T) &= \mathbf{Q}_k \delta_{k-j} \\ E(v_k v_j^T) &= \mathbf{R}_k \delta_{k-j} \end{aligned}\right\} \text{Not correlated with time}$$

$$E(w_k v_j^T) = 0$$

Process noise and Measurement noise are independant

Augmented Prediction Sigma Points:

$$x_{k-1}^a = \begin{bmatrix} \hat{x}_{k-1}^+ \\ w_k \end{bmatrix}$$

$$P_{k-1}^a = \begin{bmatrix} P_{k-1}^+ & 0 \\ 0 & Q_k \end{bmatrix}$$

$$x_{k-1}^{a(0)} = x_{k-1}^a$$
$$x_{k-1}^{a(i)} = x_{k-1}^a + \left(\sqrt{(n+\kappa)P_{k-1}^a}\right)_i$$
$$x_{k-1}^{a(n+i)} = x_{k-1}^a - \left(\sqrt{(n+\kappa)P_{k-1}^a}\right)_i$$
$$i = 1, \ldots, n$$

Prediction Step:

$$\hat{x}_k^{(i)} = f(\hat{x}_{k-1}^{(i)}, u_k, w_k^{(i)})$$

$$\hat{x}_k^- = \sum_{i=0}^{2n} W^{(i)} \hat{x}_k^{(i)}$$

$$P_k^- = \sum_{i=0}^{2n} W^{(i)} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)^T$$

$$W^{(0)} = \frac{\kappa}{n+\kappa}$$

$$W^{(i)} = \frac{1}{2(n+\kappa)} \quad i = 1, \cdots, 2n$$

Technitute_

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Unscented Kalman Filter Summary

Augmented Update Sigma Points:

$$x_k^a = \begin{bmatrix} \hat{x}_k^- \\ v_k \end{bmatrix}$$

$$P_k^a = \begin{bmatrix} P_k^- & 0 \\ 0 & R_k \end{bmatrix}$$

$$x_k^{a(0)} = x_k^a$$

$$x_k^{a(i)} = x_k^a + \left(\sqrt{(n+\kappa)P_k^a}\right)_i$$

$$x_k^{a(n+i)} = x_k^a - \left(\sqrt{(n+\kappa)P_k^a}\right)_i$$

$$i = 1, \ldots, n$$

Measurement Innovation:

$$\hat{z}_k^{(i)} = h(\hat{x}_k^{(i)}, v_k^{(i)})$$

$$\hat{z}_k = \sum_{i=0}^{2n} W^{(i)} \hat{z}_k^{(i)}$$

$$\nu_k = z_k - \hat{z}_k$$

$$S_k = \sum_{i=0}^{2n} W^{(i)} \left(\hat{z}_k^{(i)} - \hat{z}_k\right)\left(\hat{z}_k^{(i)} - \hat{z}_k\right)^T$$

$$P_{xz} = \sum_{i=0}^{2n} W^{(i)} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{z}_k^{(i)} - \hat{z}_k\right)^T$$

Update:

$$\mathbf{K}_k = \mathbf{P}_{xz}\mathbf{S}_k^{-1}$$

$$\hat{x}_k^+ = \hat{x}_k^- + \mathbf{K}_k \nu_k$$

$$\mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T$$

Technitute_

Advanced Kalman Filtering and Sensor Fusion

Unscented Kalman Filter Summary

Discrete - Time Unscented Kalman Filter Algorithm

1. The system and measurement equations are given in the following form:

$$\begin{aligned} x_k &= \mathbf{f}(x_{k-1}, u_k, w_k) \\ z_k &= \mathbf{h}(x_k, v_k) \\ w_k &\sim N(0, \mathbf{Q}_k) \\ v_k &\sim N(0, \mathbf{R}_k) \end{aligned}$$

2. The filter is initialised as follows:

$$\begin{aligned} \hat{x}_0^+ &= E[x_0] \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned}$$

3. For each time step k = 1,2, …, perform the follow prediction/update steps:

   a. Calculate the prediction sigma points for time k - 1 using:

$$x_{k-1}^a = \begin{bmatrix} \hat{x}_{k-1}^+ \\ w_k \end{bmatrix}$$

$$P_{k-1}^a = \begin{bmatrix} P_{k-1}^+ & 0 \\ 0 & Q_k \end{bmatrix}$$

$$\begin{aligned} x_{k-1}^{a(0)} &= x_{k-1}^a \\ x_{k-1}^{a(i)} &= x_{k-1}^a + \left(\sqrt{(n+\kappa)P_{k-1}^a}\right)_i \\ x_{k-1}^{a(n+i)} &= x_{k-1}^a - \left(\sqrt{(n+\kappa)P_{k-1}^a}\right)_i \end{aligned}$$

$$i = 1, \ldots, n$$

Unscented Kalman Filter Summary

Discrete - Time Unscented Kalman Filter Algorithm (Cont)

b. Calculate the prediction for time step k -1 to timestep k:

$$\hat{x}_k^{(i)} = f(\hat{x}_{k-1}^{(i)}, u_k, w_k^{(i)}) \qquad W^{(0)} = \frac{\kappa}{n+\kappa}$$

$$\hat{x}_k^- = \sum_{i=0}^{2n} W^{(i)} \hat{x}_k^{(i)} \qquad W^{(i)} = \frac{1}{2(n+\kappa)} \quad i = 1, \cdots, 2n$$

$$P_k^- = \sum_{i=0}^{2n} W^{(i)} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)^T$$

c. Calculate the measurement model sigma points for time k using:

$$x_k^a = \begin{bmatrix} \hat{x}_k^- \\ v_k \end{bmatrix} \qquad P_k^a = \begin{bmatrix} P_k^- & 0 \\ 0 & R_k \end{bmatrix}$$

$$x_k^{a(0)} = x_k^a$$

$$x_k^{a(i)} = x_k^a + \left(\sqrt{(n+\kappa)P_k^a}\right)_i$$

$$x_k^{a(n+i)} = x_k^a - \left(\sqrt{(n+\kappa)P_k^a}\right)_i$$

$$i = 1, \ldots, n$$

d. Calculate the innovation, innovation covariance and measurement cross covariance:

$$\hat{z}_k^{(i)} = h(\hat{x}_k^{(i)}, v_k^{(i)})$$

$$\hat{z}_k = \sum_{i=0}^{2n} W^{(i)} \hat{z}_k^{(i)}$$

$$\nu_k = z_k - \hat{z}_k$$

$$S_k = \sum_{i=0}^{2n} W^{(i)} \left(\hat{z}_k^{(i)} - \hat{z}_k\right)\left(\hat{z}_k^{(i)} - \hat{z}_k\right)^T$$

$$P_{xz} = \sum_{i=0}^{2n} W^{(i)} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{z}_k^{(i)} - \hat{z}_k\right)^T$$

Technitute_

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Unscented Kalman Filter Summary

Discrete - Time Unscented Kalman Filter Algorithm (Cont)

e. Perform the Kalman Filter state and covariance measurement update step calculations:

$$
\begin{aligned}
\mathbf{K}_k &= \mathbf{P}_{xz}\mathbf{S}_k^{-1} \\
\hat{x}_k^{+} &= \hat{x}_k^{-} + \mathbf{K}_k \nu_k \\
\mathbf{P}_k^{+} &= \mathbf{P}_k^{-} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^{T}
\end{aligned}
$$