ACF of SARIMA processes

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Examine ACF of a SARIMA model in simulation
- Examine ACF of a SARIMA model in theory

Example - $SARIMA(0,0,1,0,0,1)_{12}$

$$X_t = (1 + \Theta_1 B^{12})(1 + \theta_1 B)Z_t$$

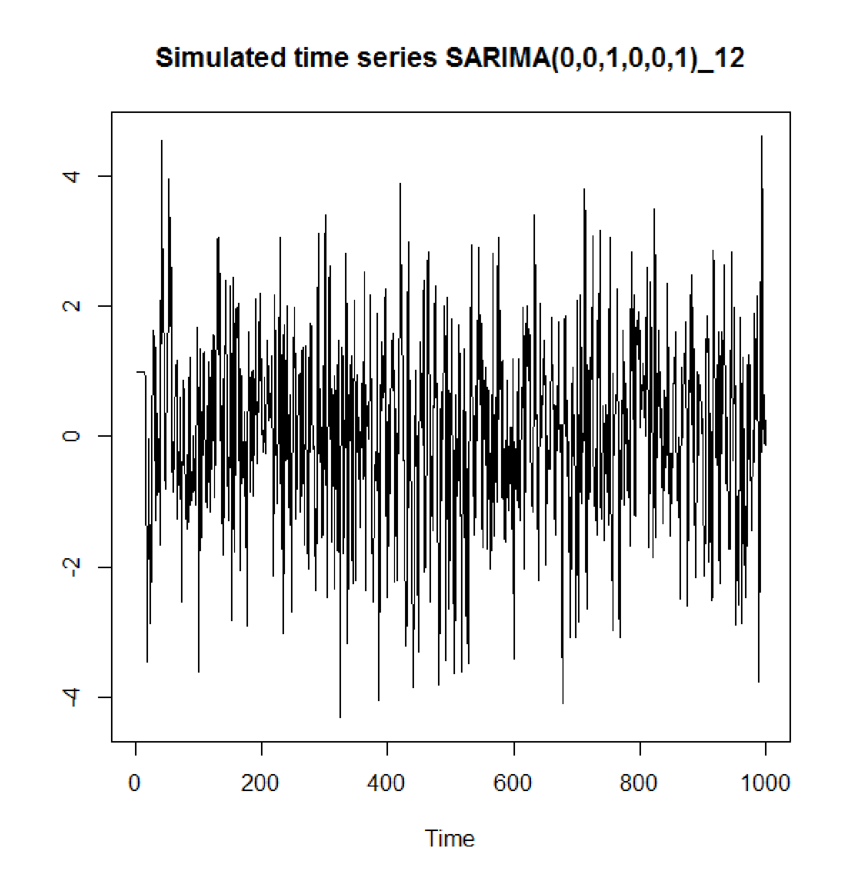
Thus

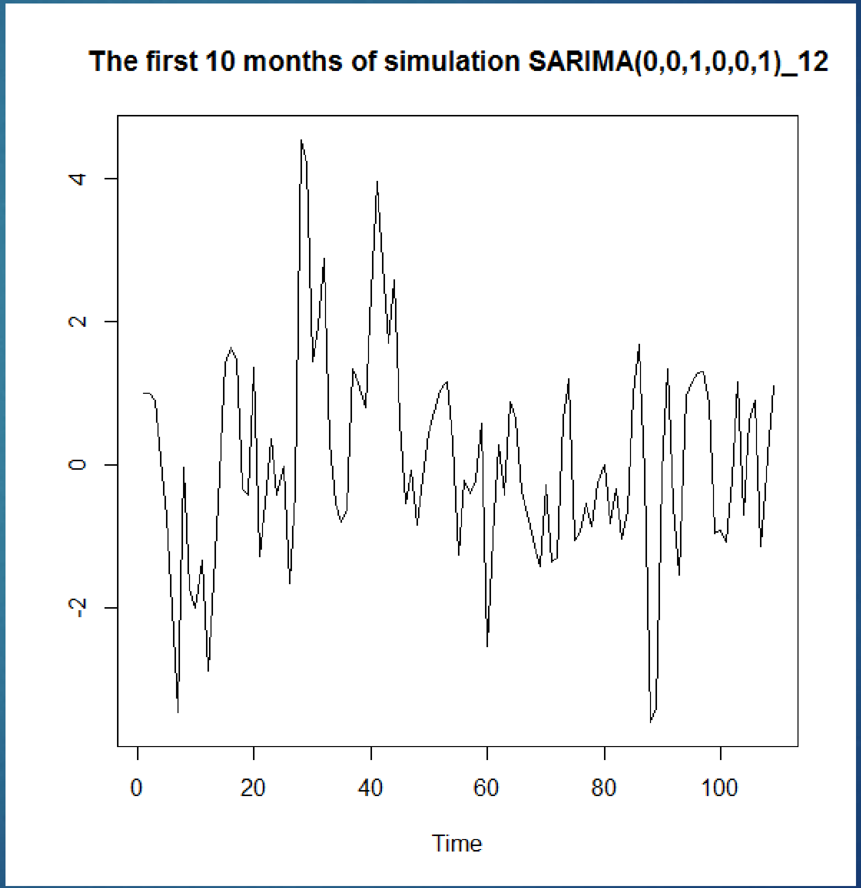
$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

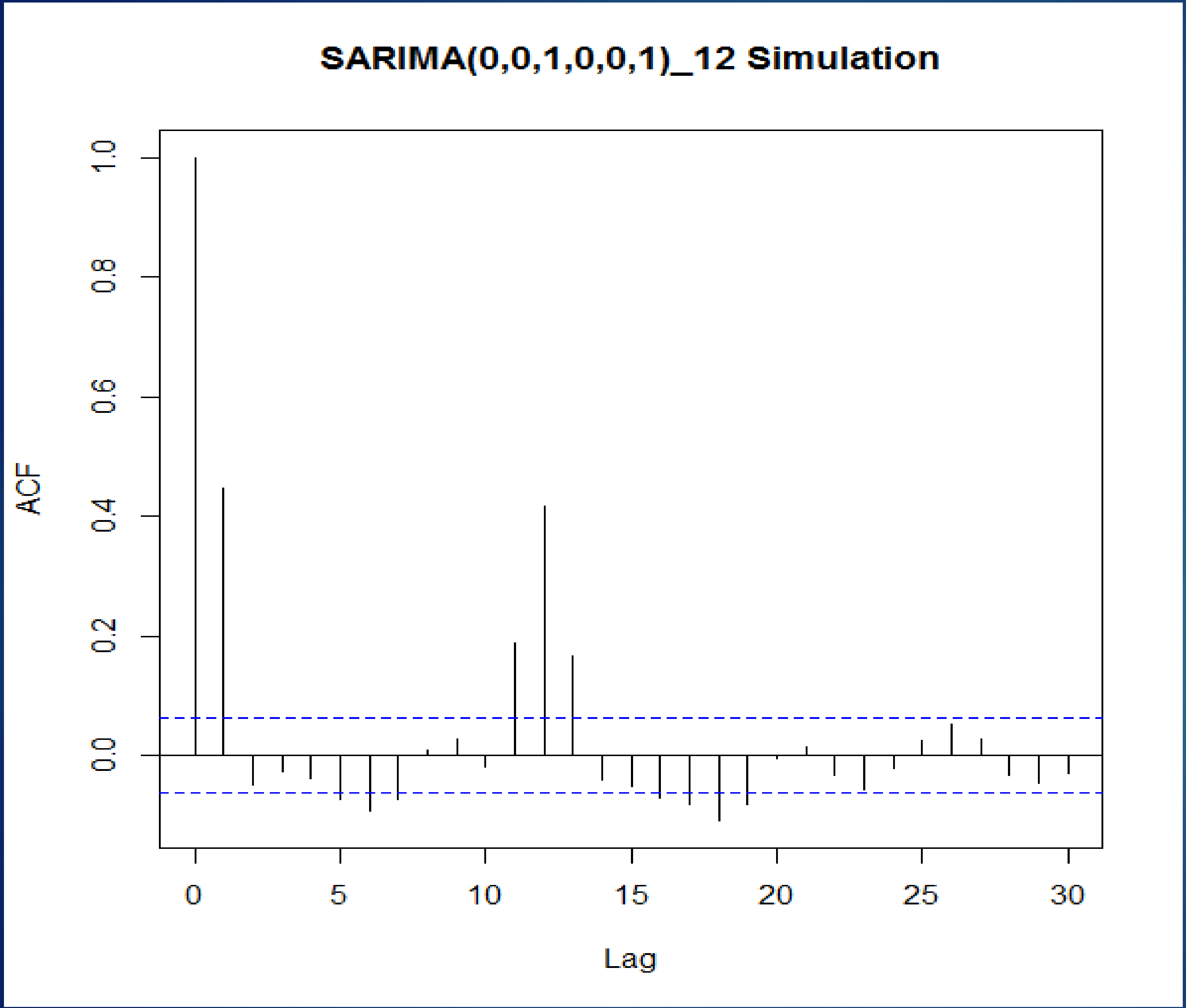
Choose $\theta_1 = 0.7, \Theta_1 = 0.6$, then

$$X_t = Z_t + 0.7\, Z_{t-1} + 0.6\, Z_{t-12} + 0.42\, Z_{t-13}$$

Simulation


Simulated time series SARIMA(0,0,1,0,0,1)_12
-4
-2
0
2
4
0
200
400
600
800
1000
Time


The first 10 months of simulation SARIMA(0,0,1,0,0,1)_12
-2
0
2
4
0
20
40
60
80
100
Time


SARIMA(0,0,1,0,0,1)_12 Simulation
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
Lag

Example - $SARIMA(0,0,1,0,0,1)_{12}$

$$X_t = (1 + \Theta_1 B^{12})(1 + \theta_1 B)Z_t$$

Thus

$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

Autocovariance function: $\gamma(k)$

$$\gamma(0) = Cov(X_t, X_t) = Var(X_t)$$

$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

$$Var(X_t) = \sigma_Z^2 + \theta_1^2 \sigma_Z^2 + \Theta_1^2 \sigma_Z^2 + \theta_1^2 \Theta_1^2 \sigma_Z^2$$

$$\gamma(0) = (1 + \theta_1^2)(1 + \Theta_1^2)\sigma_Z^2$$

$\gamma(1)$

$$\gamma(1) = Cov(X_t, X_{t-1})$$

$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

$$X_{t-1} = Z_{t-1} + \theta_1 Z_{t-2} + \Theta_1 Z_{t-13} + \theta_1 \Theta_1 Z_{t-14}$$

$$\gamma(1) = \theta_1 \sigma_Z^2 + \theta_1 \Theta_1^2 \sigma_Z^2$$

$$\gamma(1) = \theta_1 (1 + \Theta_1^2) \sigma_Z^2$$

ACF: $\rho(1)$

$$\gamma(1) = \theta_1(1 + \Theta_1^2)\sigma_Z^2$$

$$\gamma(0) = (1 + \theta_1^2)(1 + \Theta_1^2)\sigma_Z^2$$

$$\rho(1) = \frac{\gamma(1)}{\gamma(0)} = \frac{\theta_1}{1 + \theta_1^2} \leq \frac{1}{2}$$

Since $(\theta_1 - 1)^2 \geq 0$

$\gamma(2)$

$$\gamma(2) = Cov(X_t, X_{t-2})$$

$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

$$X_{t-2} = Z_{t-2} + \theta_1 Z_{t-3} + \Theta_1 Z_{t-14} + \theta_1 \Theta_1 Z_{t-15}$$

$$\gamma(2) = 0$$

since $Z_t's$ are independent.

Thus

$$\rho(2) = 0$$

ACF

$$\rho(i) = 0$$

when $i = 2,3,\ldots,10.$

$\gamma(11)$, $\rho(11)$

$$\gamma(11) = Cov(X_t, X_{t-11})$$

$$X_t = Z_t + \theta_1 Z_{t-1} + \Theta_1 Z_{t-12} + \theta_1 \Theta_1 Z_{t-13}$$

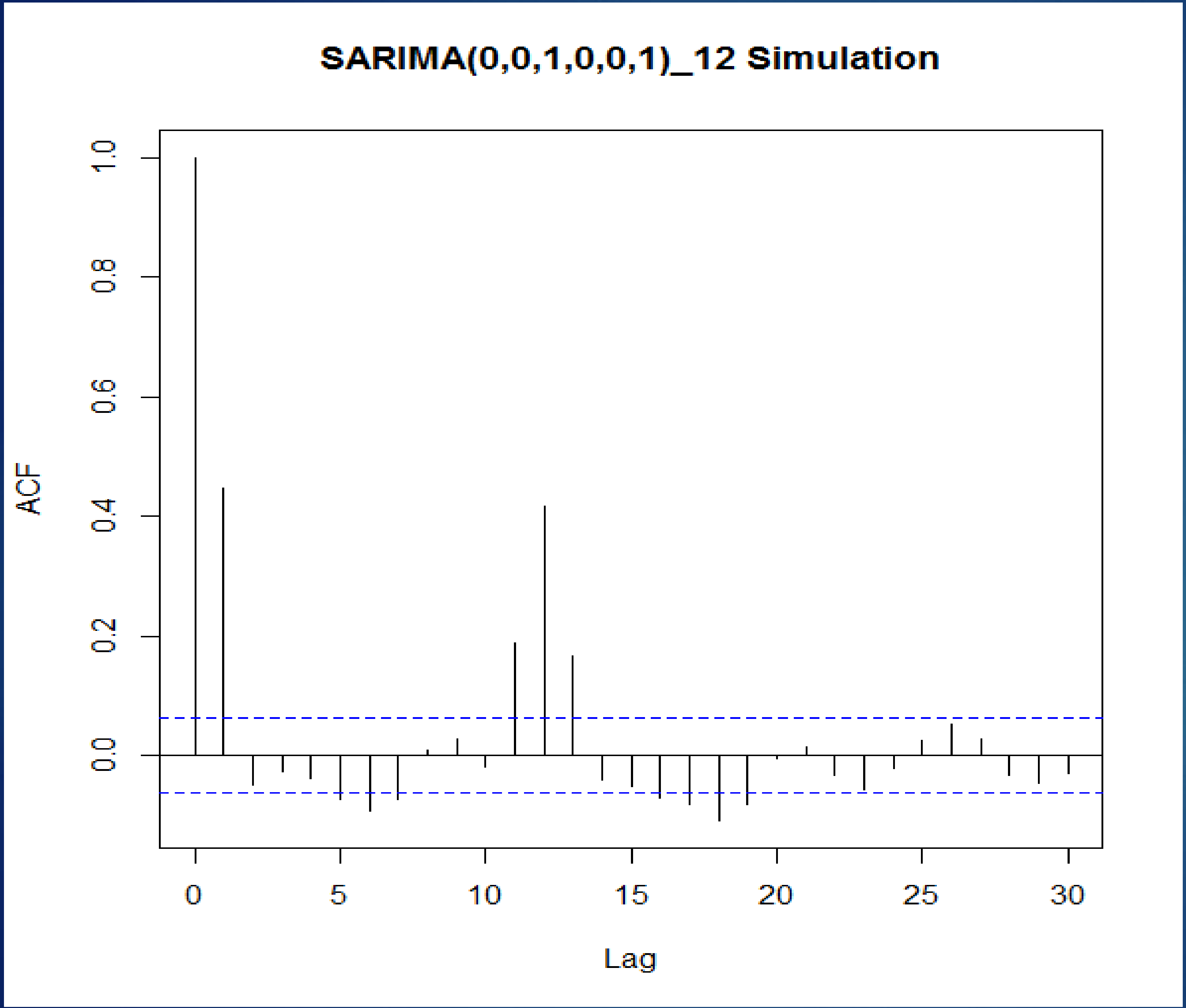
$$X_{t-11} = Z_{t-11} + \theta_1 Z_{t-12} + \Theta_1 Z_{t-23} + \theta_1 \Theta_1 Z_{t-24}$$

$$\gamma(11) = \theta_1 \Theta_1 \sigma_Z^2$$

$$\rho(11) = \frac{\gamma(11)}{\gamma(0)} = \frac{\theta_1 \Theta_1}{(1+\theta_1^2)(1+\Theta_1^2)} \neq 0$$

But

$$0 < \rho(11) \leq \frac{1}{4}$$


SARIMA(0,0,1,0,0,1)_12 Simulation
ACF
Lag
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30

What We've Learned

- ACF of a SARIMA model in simulation
- ACF of a SARIMA model in theory