Parameter estimation: Recruitment

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- To fit an AR(p) model to recruitment (number of new fish) for a period of 453 months ranging over the years 1950-1987.
- Use Yule-Walker equations in matrix form to estimate parameters of the fitted model

rec {astsa}

- Recruitment (number of new fish) for a period of 453 months ranging over the years 1950-1987.
- Monthly time series
- Source: “astsa” package

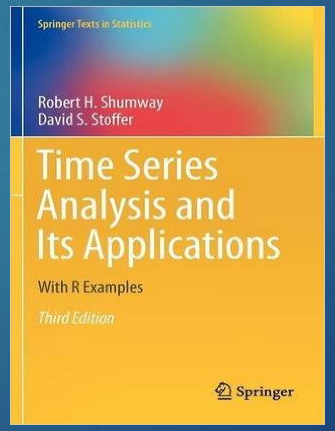
Shumway, R.H. and Stoffer, D.S. (2000)

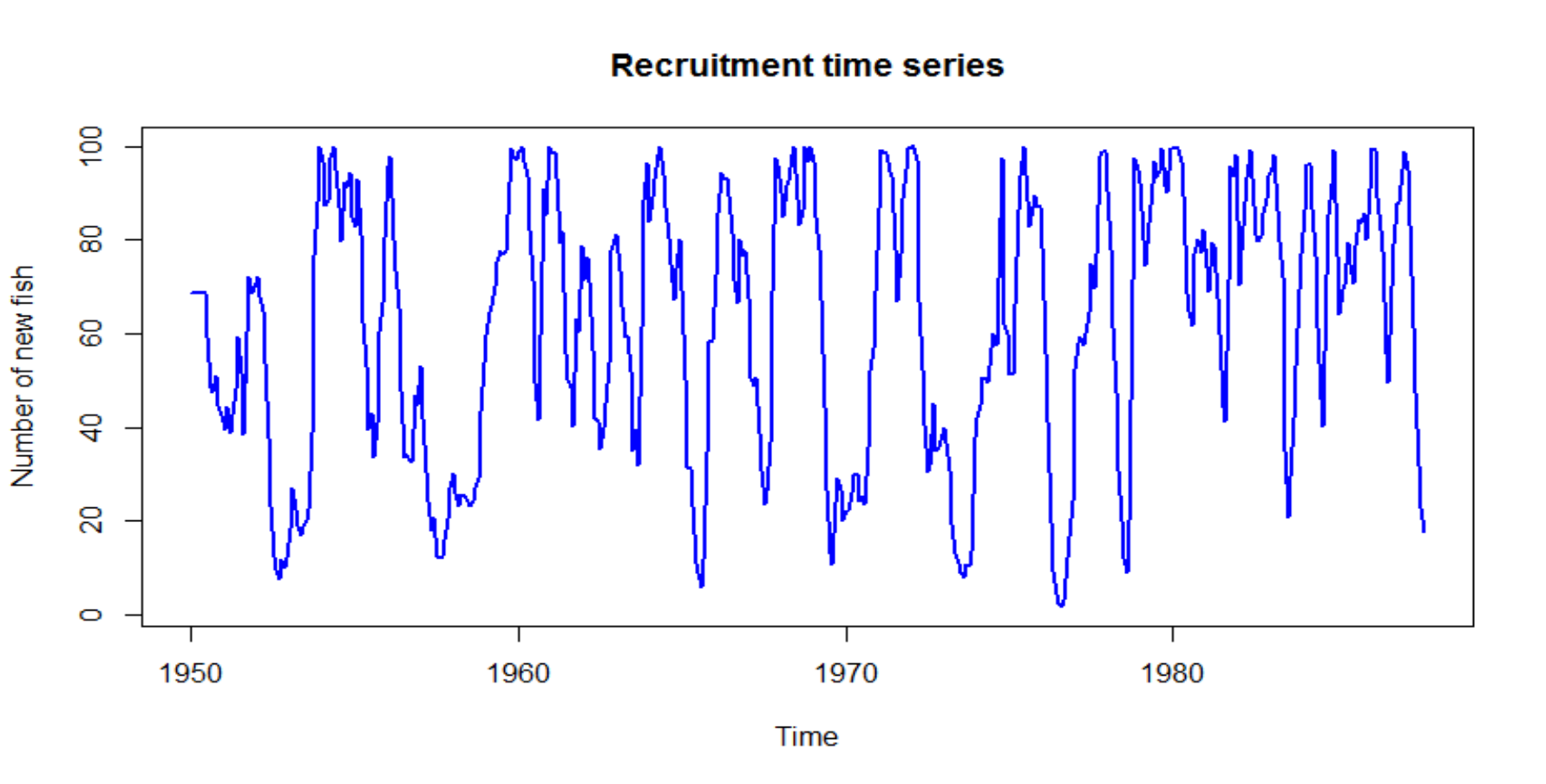
Time Series Analysis and its Applications

With R examples

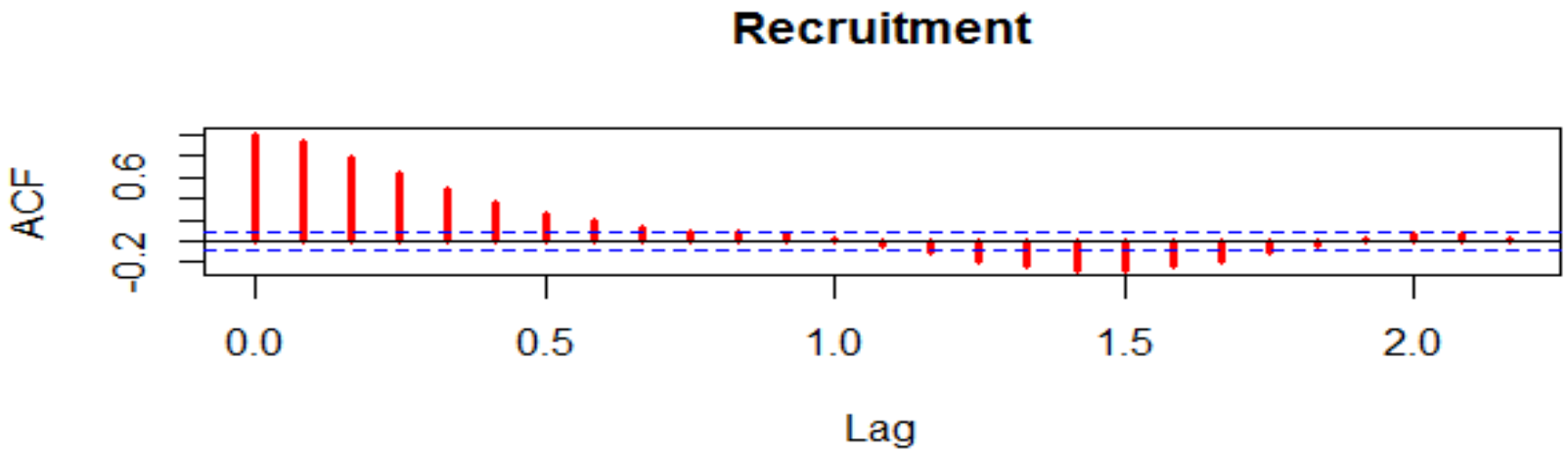
Third Edition

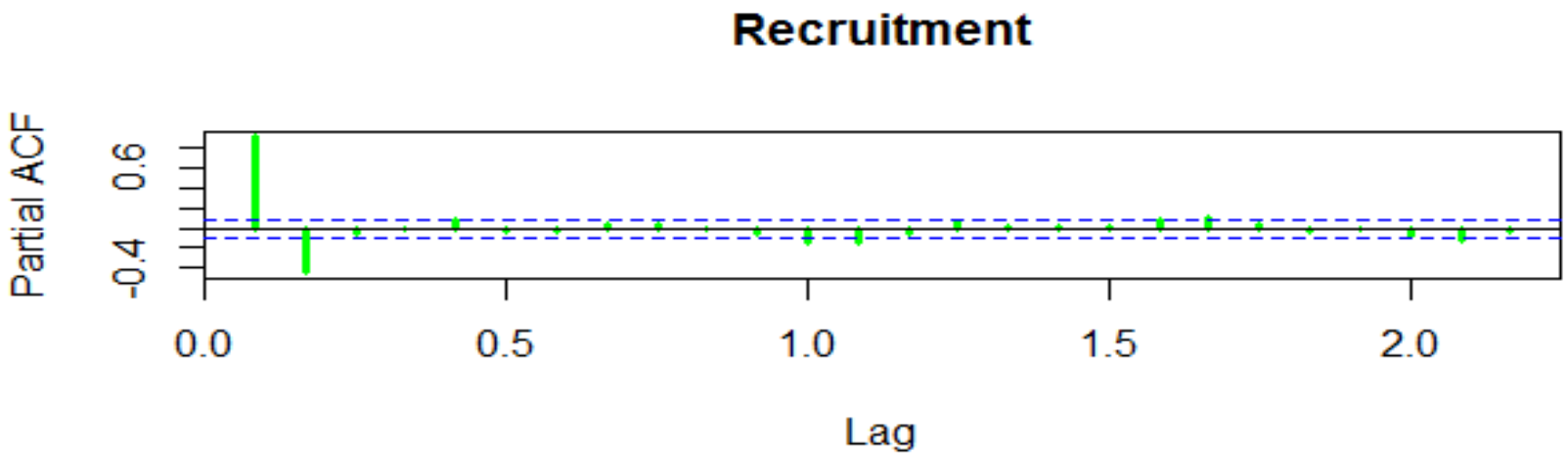
Springer




Recruitment time series
Number of new fish
0
20
40
60
80
100
1950
1960
1970
1980
Time

ACF and PACF


Recruitment
ACF
-0.2
0.6
0.0
0.5
1.0
1.5
2.0
Lag


Recruitment
Partial ACF
-0.4
0.6
0.0
0.5
1.0
1.5
2.0
Lag

The parsimony principle

- Choose ‘simplest explanation that fits the evidence’
- Simplest of competing theories is to be preferred
- PACF $\Longrightarrow$ AR(2)
- Yule-Walker equations in matrix form

Code

- ar.process=rec-mean(rec)

$$X_t - \mu$$

- p=2
- Yule-Walker equations: $\hat{R}\hat{\phi} = \hat{b}$
- Sample autocorrelation coefficients, vector r

```
for(i in 1:p+1){
        r[i-1]<-acf(ar.process, plot=F)$acf[i]
        }
```

Matrix $\hat{R}$

$$\begin{bmatrix} 1 & r_1 & r_2 & & r_{p-1} \\ r_1 & 1 & r_1 & \cdots & r_{p-2} \\ r_2 & r_1 & 1 & & r_{p-3} \\ & \vdots & & \ddots & \vdots \\ r_{p-2} & r_{p-3} & r_{p-4} & & r_1 \\ r_{p-1} & r_{p-2} & r_{p-3} & \cdots & 1 \end{bmatrix}$$

Realize

$$\hat{R}(i,j) = \hat{R}_{ij} = r_{|i-j|}$$

- R=matrix(1,p,p) # matrix of dimension p by p, with entries all 1's.

- ```
  for(i in 1:p){
      for(j in 1:p){
          if(i!=j)
              R[i,j]=r[abs(i-j)]
          }
      }
  ```
  ```

- # b-column vector on the right

```
b=matrix(,p,1)# b- column vector with no entries
for(i in 1:p){
        b[i,1]=r[i]
        }
```

- # solve(R,b) solves Rx=b, and gives x=R^(-1)b vector

```
phi.hat=NULL
for(i in 1:p){
        phi.hat[i]=solve(R,b)[i,1]
        }
```

Model

$$X_t - \bar{x} = \hat{\phi}_1(X_{t-1} - \bar{x}) + \hat{\phi}_2(X_{t-2} - \bar{x}) + \cdots + \hat{\phi}_p(X_{t-p} - \bar{x}) + Z_t$$

Thus

$$X_t = \hat{\phi}_0 + \hat{\phi}_1 X_{t-1} + \hat{\phi}_2 X_{t-2} + \cdots + \hat{\phi}_p X_{t-p} + Z_t$$

where

$$\hat{\phi}_0 = \bar{x}(1 - \sum_{i=1}^{p} \hat{\phi}_i)$$

$p = 2$

Fitted model is

$$X_t = 7.033036 + 1.331587\, X_{t-1} - 0.4445447\, X_{t-2} + Z_t$$

$$Z_t \sim Normal\ (0, 94.17131)$$

What We've Learned

- Fitting an AR(p=2) model to Recruitment (number of new fish) from 'astsa' package using Yule-Walker equations in matrix form