Estimating model parameters – AR(3) Simulation

PRACTICAL TIME SERIES ANALYSIS

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Objectives

- Estimate model parameters of a simulated AR(3) process using Yule-Walker equations in matrix form

AR(2) process (with mean zero)

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + Z_t$$

where

$$Z_t \sim Normal\ (0, \sigma_Z^2)$$

We simulate this process for

$$\phi_1 = \frac{1}{3}, \phi_2 = \frac{1}{2}, \phi_3 = \frac{7}{100}, \sigma_Z = 4$$

Yule –Walker equaitons

We (will) estimate coefficients of the model

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + Z_t$$

by first finding r_1, r_2 using acf() routine, then solving the system of equations

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 & r_1 & r_2 \\ r_1 & 1 & r_1 \\ r_2 & r_1 & 1 \end{bmatrix} \begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \hat{\phi}_3 \end{bmatrix}$$

σ_Z Estimation

Yule – Walker estimator for σ_Z^2

$$\hat{\sigma}_Z^2 = c_0\left(1 - \sum_{i=1}^{p} \phi_i r_i\right)$$

Results (set.seed(2017))

- $\mathrm{n}= 100000$
- $\phi_1 \approx 0.3381245$
- $\phi_2 \approx 0.4984999$
- $\phi_3 \approx 0.06849712$
- $\sigma_Z^2 \approx 15.979$

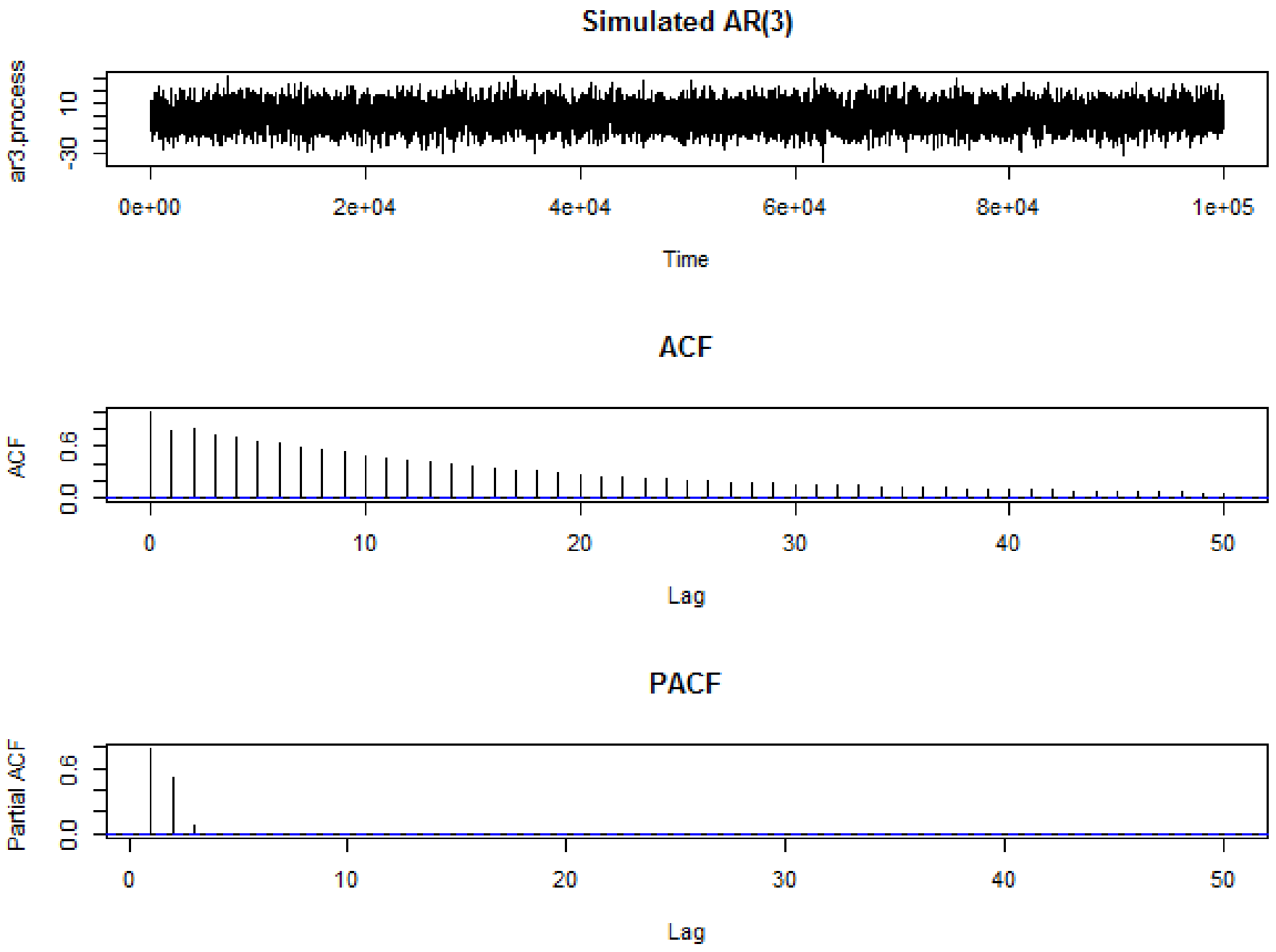
Actual Model

$$X_t = 0.\overline{3}\, X_{t-1} + 0.5X_{t-2} + 0.07X_{t-3} + Z_t, \qquad Z_t \sim N(0,16)$$

Fitted model

$$X_t = 0.3381245\, X_{t-1} + 0.4984999\, X_{t-2} + 0.06849712\, X_{t-3} + Z_t,$$

$$Z_t \sim N(0,15.979)$$


Simulated AR(3)
ar3.process
-30
10
0e+00
2e+04
4e+04
6e+04
8e+04
1e+05
Time
ACF
ACF
0.0
0.6
0
10
20
30
40
50
Lag
PACF
Partial ACF
0.0
0.6
0
10
20
30
40
50
Lag

What We've Learned

- Estimating model parameters of a simulated AR(3) process using Yule-Walker equations in a matrix form