



# *Random Walk*

PRACTICAL TIME SERIES ANALYSIS

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# Objectives

- ▶ Get familiar with the random walk model
- ▶ Simulate a random walk in  $\mathbb{R}$
- ▶ Obtain the correlogram of a random walk
- ▶ See the difference operator in action

# Model

Location at previous step  
(or price of the stock yesterday)

Location at time  $t$   
(or a price of a stock today)

$$X_t = X_{t-1} + Z_t$$

$Z_t \sim \text{Normal}(\mu, \sigma^2)$

White noise  
(residual)

$$X_0 = 0$$



$$X_1 = Z_1$$



$$X_2 = Z_1 + Z_2$$



$$X_t = \sum_{i=1}^t Z_i$$



...

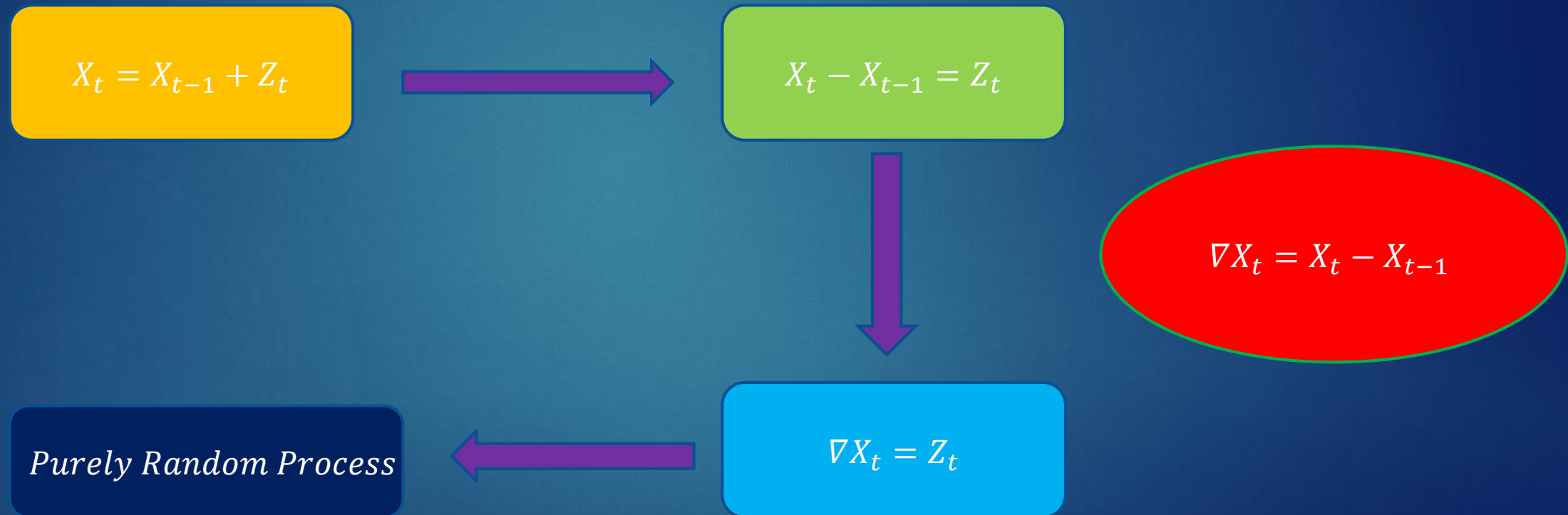
$$E[X_t] = E\left[\sum_{i=1}^t Z_i\right] = \sum_{i=1}^t E[Z_i] = \mu t$$

$$\text{Var}[X_t] = \text{Var}\left[\sum_{i=1}^t Z_i\right] = \sum_{i=1}^t \text{Var}[Z_i] = \sigma^2 t$$

# Simulation

- ▶  $X_1 = 0$
- ▶  $Z_t \sim \text{Normal}(0, 1)$
- ▶  $X_t = X_{t-1} + Z_t$  for  $t = 2, 3, \dots, 1000$
- ▶ Plot and ACF

# Removing the trend



# Difference operator

- ▶ `diff()` to remove the trend
- ▶ Plot and ACF differenced time series

# What We've Learned

- ▶ Random Walk model
- ▶ How to simulate a random walk in R
- ▶ How to get stationary time series from a random walk using `diff()` operator