Random Walk

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Get familiar with the random walk model
- Simulate a random walk in R
- Obtain the correlogram of a random walk
- See the difference operator in action

Model

Location at
previous step
(or price of the
stock yesterday)

Location at
time t
(or a price of a
stock today)

$$X_t = X_{t-1} + Z_t$$

$$Z_t \sim \ Normal\ (\mu, \sigma^2)$$

White noise
(residual)

$$X_0 = 0$$

$$X_1 = Z_1$$

$$X_2 = Z_1 + Z_2$$

$$\dots$$

$$X_t = \sum_{i=1}^{t} Z_i$$

$$E[X_t] = E\left[\sum_{i=1}^{t} Z_i\right] = \sum_{i=1}^{t} E[Z_i] = \mu t$$

$$Var[X_t] = Var\left[\sum_{i=1}^{t} Z_i\right] = \sum_{i=1}^{t} Var[Z_i] = \sigma^2 t$$

Simulation

- $X_1 = 0$
- $Z_t \sim Normal\ (0, 1)$
- $X_t = X_{t-1} + Z_t$ for $t = 2,3, \dots, 1000$
- Plot and ACF

Removing the tend

$$X_t = X_{t-1} + Z_t$$

$$X_t - X_{t-1} = Z_t$$

$$\nabla X_t = Z_t$$

Purely Random Process

$$\nabla X_t = X_t - X_{t-1}$$

Difference operator

- diff() to remove the trend
- Plot and ACF differenced time series

What We've Learned

- Random Walk model
- How to simulate a random walk in R
- How to get stationary time series from a random walk using diff() operator